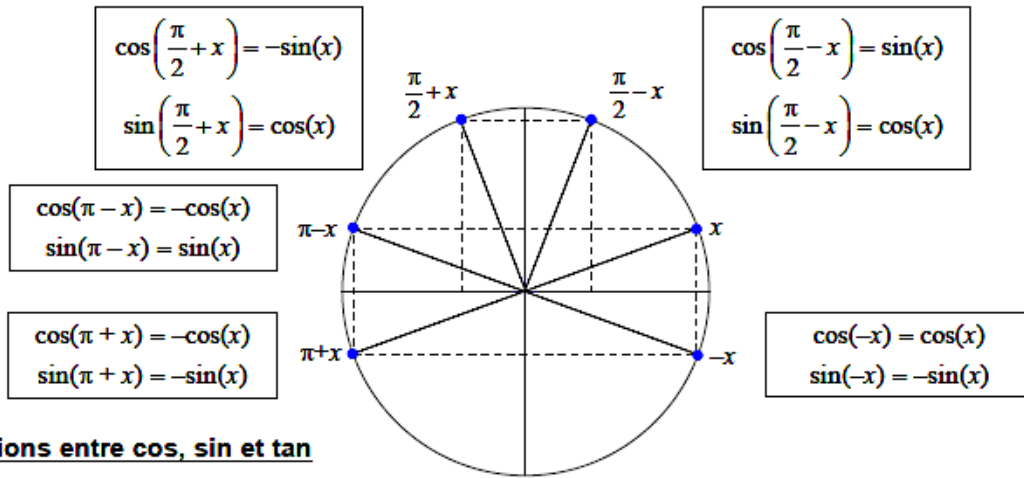


Formulaire trigonométrique

Une lecture efficace du cercle trigonométrique permet de retrouver les relations suivantes :



Relations entre cos, sin et tan

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \frac{1}{\cos^2(x)}$$

Formules d'addition

$$\cos(a - b) = \cos(a) \cos(b) + \sin(a) \sin(b)$$

$$\cos(a + b) = \cos(a) \cos(b) - \sin(a) \sin(b)$$

$$\sin(a - b) = \sin(a) \cos(b) - \cos(a) \sin(b)$$

$$\sin(a + b) = \sin(a) \cos(b) + \cos(a) \sin(b)$$

$$\tan(a - b) = \frac{\tan(a) - \tan(b)}{1 + \tan(a) \tan(b)}$$

$$\tan(a + b) = \frac{\tan(a) + \tan(b)}{1 - \tan(a) \tan(b)}$$

Formules de duplication

$$\cos(2a) = \cos^2(a) - \sin^2(a) = 2 \cos^2(a) - 1 = 1 - 2 \sin^2(a) \quad \sin(2a) = 2 \sin(a) \cos(a) \quad \tan(2a) = \frac{2 \tan(a)}{1 - \tan^2(a)}$$

Extensions : $\cos(3a) = 4 \cos^3(a) - 3 \cos(a)$

$\sin(3a) = 3 \sin(a) - 4 \sin^3(a)$

$\tan(3a) = \frac{3 \tan(a) - \tan^3(a)}{1 - 3 \tan^2(a)}$

Au delà, utiliser la formule de Moivre.

Formules de linéarisation

$$\cos^2(a) = \frac{1 + \cos(2a)}{2}$$

$$\sin^2(a) = \frac{1 - \cos(2a)}{2}$$

$$\tan^2(a) = \frac{1 - \cos(2a)}{1 + \cos(2a)}$$

Extensions : $\cos^3(a) = \frac{\cos(3a) + 3 \cos(a)}{4}$

$\sin^3(a) = \frac{-\sin(3a) + 3 \sin(a)}{4}$

$\tan^3(a) = \frac{-\sin(3a) + 3 \sin(a)}{\cos(3a) + 3 \cos(a)}$

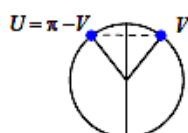
Au delà, utiliser les formules d'Euler. Les formules d'Euler permettent également de montrer que :

$$\cos(a) \cos(b) = \frac{1}{2} [\cos(a - b) + \cos(a + b)] \quad \cos(a) \sin(b) = \frac{1}{2} [\sin(a + b) - \sin(a - b)] \quad \sin(a) \sin(b) = \frac{1}{2} [\cos(a - b) - \cos(a + b)]$$

$$\cos(p) + \cos(q) = 2 \cos\left(\frac{p+q}{2}\right) \cos\left(\frac{p-q}{2}\right) \quad \cos(p) - \cos(q) = -2 \sin\left(\frac{p+q}{2}\right) \sin\left(\frac{p-q}{2}\right)$$

$$\sin(p) + \sin(q) = 2 \sin\left(\frac{p+q}{2}\right) \cos\left(\frac{p-q}{2}\right) \quad \sin(p) - \sin(q) = 2 \sin\left(\frac{p-q}{2}\right) \cos\left(\frac{p+q}{2}\right)$$

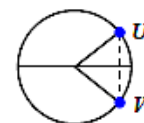
Résolution d'équations trigonométriques



$$\cos(U) = \cos(V) \Leftrightarrow (U = V [2\pi] \text{ ou } U = -V [2\pi])$$

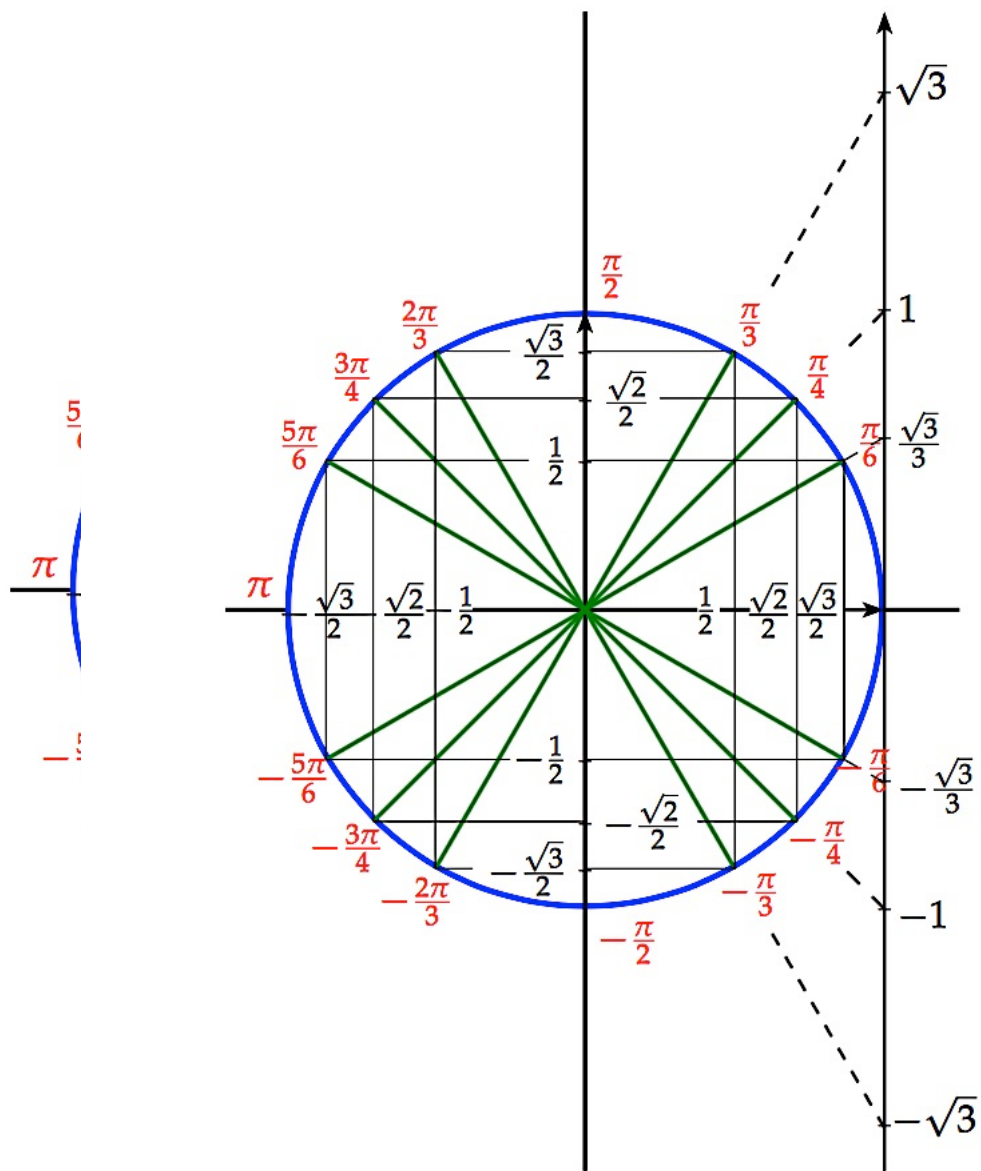
$$\sin(U) = \sin(V) \Leftrightarrow (U = V [2\pi] \text{ ou } U = \pi - V [2\pi])$$

$$\tan(U) = \tan(V) \Leftrightarrow U = V [\pi]$$



Expression du cosinus, sinus et tangente en fonction de la tangente de l'angle moitié

Si $t = \tan\left(\frac{a}{2}\right)$, on a : $\cos(a) = \frac{1-t^2}{1+t^2}$; $\sin(a) = \frac{2t}{1+t^2}$; $\tan(a) = \frac{2t}{1-t^2}$



Valeurs remarquables

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$		0