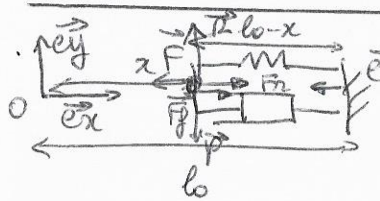
Modélisation mécanique du haut parleur



1) Le PFD sur M dans RTSG donne en proj sur Ox ($\vec P + \vec R = 0$ verticalement)

$$m\ddot x = -k(\overbrace{l_0 - x}^{\ell} - l_0)(\overset{-\vec{e_x}}{\vec{e_1}}) - \lambda \dot x + F(t)$$

$\Leftrightarrow \; m\ddot x + \lambda \dot x + kx = F(t)$ (1)

2) (1) $\Leftrightarrow \; \ddot x + \frac{\omega_0}{Q}\dot x + \omega_0^2 x = \frac{F(t)}{m}$, avec $\boxed{\omega_0 = \sqrt{\frac{k}{m}} = 1200 \text{ rad/s}}$

$\frac{\omega_0}{Q} = \frac{\lambda}{m} \Rightarrow \boxed{Q = \frac{\sqrt{mk}}{\lambda}}$

On a $Q = \frac{1}{\sqrt2} \Leftrightarrow \boxed{\lambda = \sqrt{2mk} = 17 \text{ kg s}^{-1}}$

♡ unité ? → $\ddot x + \frac{\lambda}{m}\dot x$ homogène ⇒ $\frac{\lambda}{m}$ en s^{-1}

3) En complexes : $\underline{x}\left(\omega_0^2 - \omega^2 + j\omega\frac{\omega_0}{Q}\right) = \frac{\underline{F}(t)}{m} = \frac{K_0 I_m}{m}$

$$\Rightarrow \underline{x} = \frac{K_0 I_m}{m\left(\omega_0^2-\omega^2+j\omega\frac{\omega_0}{Q}\right)}$$

4) D'où

$$\varphi = \arg(\underline{x}) = -\operatorname{Arctan}\frac{\omega\omega_0}{Q(\omega_0^2-\omega^2)} \overset{AN}{=} -0{,}27 \text{ rad}$$

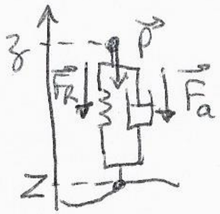
$$x_m = \frac{K_0 I_m/m}{\sqrt{(\omega_0^2-\omega^2)^2 + \left(\frac{\omega\omega_0}{Q}\right)^2}} \overset{AN}{=} 0{,}52 \text{ mm}$$

Oscillation d'1 remorque

1) Alors poids et Fressort se compensent, le ressort étant comprimé de x, tel que $mg = kx \Rightarrow x = \frac{mg}{k}$

$$\Rightarrow \boxed{l_1 = l_0 - \frac{mg}{k} = 53 \text{ cm}}$$

2)



PFD sur M en proj / Oz :

$$m\ddot z = -k(z - Z - l_0) - mg - \lambda(\dot z - \dot Z)$$
$$= -k\Big(z - \underbrace{\left(l_0 - \frac{mg}{k}\right)}_{l_1}\Big) - \lambda(\dot z - \dot Z)$$

z' (par rapport à O')

soit $\ddot z' + \frac{k}{m}\dot z' + \frac{\lambda}{m} z' = \underbrace{\frac{k}{m}Z + \frac{\lambda}{m}\dot Z}_{\text{nul ici ...}}$

♡ On a $\frac{k}{m} = \omega_0^2$, $\frac{\lambda}{m} = \frac{2\sqrt{mk}}{m} = 2\omega_0 \Rightarrow Q = \frac{1}{2}$. L'équation se réécrit $\ddot z' + 2\omega_0 \dot z' + \omega_0^2 z = 0$, $\Delta' = \omega_0^2 - \omega_0^2 = 0$

$r_1 = r_2 = -\omega_0 \Rightarrow$ solution $z = (At+B)e^{-\omega_0 t}$

à $t=0$, $\left\{\begin{array}{l} z = z_0 \Rightarrow B = z_0 \\ \dot z = 0 \Rightarrow A - \omega_0 B = 0 \Rightarrow A = \omega_0 z_0 \end{array}\right\} \Rightarrow z = z_0(\omega_0 t + 1)e^{-\omega_0 t}$

3) a) $L = vT = \frac{2\pi v}{\omega} \Rightarrow \omega = \frac{2\pi v}{L}$ b) cf + haut ...

c) En complexes, $\underline{z}(-\omega^2 + 2j\omega\omega_0 + \omega_0^2) = \frac{k}{m}\underline{Z} + \frac{\lambda}{m} j\omega \underline{Z}$

module $\Rightarrow$

$$z_0 = \frac{\sqrt{\left(\frac{k}{m}\right)^2 + \left(\frac{\omega\lambda}{m}\right)^2}\; Z_0}{\sqrt{(\omega_0^2-\omega^2)^2 + (2\omega\omega_0)^2}}$$

Décrement logarithmique

classique !

1) $\ddot{x} + \frac{h}{m}\dot{x} + \frac{k}{m}x = 0$ $\quad \boxed{\omega_0 = \sqrt{\frac{k}{m}}}$, $2\xi\sqrt{\frac{k}{m}} = \frac{h}{m} \Rightarrow \boxed{\xi = \frac{h}{2\sqrt{mk}}}$

2) Eq caractéristique $\Delta' = \xi^2\omega_0^2 - \omega_0^2 < 0$ si $\boxed{\xi < 1}$

3) Solutions $r_{1/2} = -\xi\omega_0 \pm i\Omega$, $\boxed{\Omega = \omega_0\sqrt{1-\xi^2}}$

$$\Rightarrow x = e^{-\xi\omega_0 t}(A\cos\Omega t + B\sin\Omega t)$$

à $t=0$ $x=0 \Rightarrow A=0$, $\dot{x} = -\xi\omega_0 e^{-\xi\omega_0 t}B\sin\Omega t + \Omega B\cos\Omega t\, e^{-\xi\omega_0 t}$

$\dot{x}(0) = V_0 \Rightarrow B = \frac{V_0}{\Omega}$

Donc $\boxed{x = \frac{V_0}{\Omega}\sin\Omega t\, e^{-\xi\omega_0 t}}$

2.3) $\Rightarrow \frac{x(t)}{x(t+nT)} = \frac{e^{-\xi\omega_0 t}}{e^{-\xi\omega_0(t+nT)}} = e^{n\xi T\omega_0}$

Donc $\delta = \xi T\omega_0 = \frac{\xi 2\pi\omega_0}{\Omega} = \boxed{\frac{\xi 2\pi}{\sqrt{1-\xi^2}} = \delta}$

2.4 Courbe $\Rightarrow \boxed{T = 2,4\,s}$, $\boxed{\delta = \ln\left(\frac{4}{0,8}\right) \approx 1,6}$

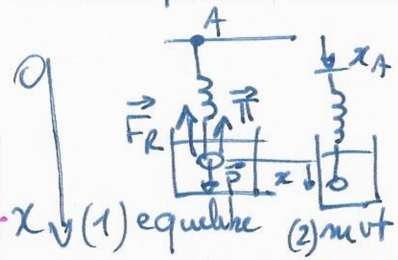
or $\delta = \frac{\xi 2\pi}{\sqrt{1-\xi^2}} \Rightarrow 1-\xi^2 = \xi^2\left(\frac{2\pi}{\delta}\right)^2 \Rightarrow \xi = \frac{1}{\sqrt{1+(2\pi/\delta)^2}} \overset{AN}{=} 0,24$

On a $\Omega = \omega_0\sqrt{1-\xi^2} = \sqrt{\frac{k}{m}(1-\xi^2)} = \frac{2\pi}{T}$

$$\Rightarrow k = \left(\frac{2\pi}{T}\right)^2\frac{m}{1-\xi^2} \overset{AN}{=} 3,6\ N\,m^{-1}$$

Bande passante en vitesse (Cours)

Savoir refaire



(1) Eq, proj sur Ox : $mg - \Pi - k(x_{eq} - l_0) = 0$

(2) mvt : $m\ddot{x} = \cancel{mg} - \cancel{\Pi} - k(\cancel{x_{eq}} + x - \cancel{l_0} - x_A) - \lambda\dot{x}$

d'après (1)

$\Rightarrow \ddot{x} + \frac{\lambda}{m}\dot{x} + \frac{k}{m}x = \frac{k}{m}x_A$

$\Leftrightarrow \ddot{x} + \frac{\omega_0}{Q}\dot{x} + \omega_0^2 x = x_A\omega_0^2$ avec $\omega_0 = \sqrt{\frac{k}{m}}$, $Q = \frac{m\omega_0}{\lambda}$

Complexes $\underline{X} = \frac{x_A\omega_0^2}{\omega_0^2 - \omega^2 + j\frac{\omega_0}{Q}\omega}$ En posant $u = \frac{\omega}{\omega_0}$,

$X = \frac{x_A\omega_0^2}{\sqrt{(1-u^2)^2 + \frac{u^2}{Q^2}}}$ $\xrightarrow{\text{dériv}}$ $V = \omega X = \frac{x_A\omega_0^3 u}{\sqrt{(1-u^2)^2 + \frac{u^2}{Q^2}}}$

$V = \frac{x_A\omega_0^2}{\sqrt{\left(u - \frac{1}{u}\right)^2 + \frac{1}{Q^2}}}$

V_{max} pour $u = 1 \Leftrightarrow \boxed{\omega = \omega_0}$

$V_{max} = Q x_A \omega_0$

"astuce" à connaître

$V = \frac{V_{max}}{\sqrt{2}} \Leftrightarrow \left(u - \frac{1}{u}\right)^2 + \frac{1}{Q^2} = \frac{2}{Q^2} \Rightarrow u - \frac{1}{u} = \pm\frac{1}{Q}$

$\Leftrightarrow u^2 \pm \frac{u}{Q} - 1 = 0$ 2 Eqs 2d degré en u, de

4 solutions $u = \pm\frac{1}{2Q} \pm \frac{\sqrt{\Delta}}{2}$ $\quad \Delta = \frac{1}{Q^2} + 4$

dont 2 positives $\left(\sqrt{\Delta} > \frac{1}{Q}\right)$

$$\left.\begin{aligned} u_1 &= -\frac{1}{2Q} + \frac{\sqrt{\Delta}}{2} \\ u_2 &= \frac{1}{2Q} + \frac{\sqrt{\Delta}}{2} \end{aligned}\right\} \Delta u = \frac{1}{Q}$$

$$\Rightarrow \Delta\omega = \frac{\omega_0}{Q}$$