

BONNER Antonin Fe036, Ex3 (91):

Ex3:

① On pose $\lfloor x \rfloor = k \in \mathbb{Z}$ $x = k + \alpha$ avec $\alpha \in [0; 1[$

$$mx = m(k + \alpha) = mk + m\alpha$$

avec la partie entière:

$$\Leftrightarrow \lfloor mx \rfloor = \lfloor mk + m\alpha \rfloor = mk + \lfloor m\alpha \rfloor$$

$$\Leftrightarrow \frac{\lfloor mx \rfloor}{m} = k + \frac{\lfloor m\alpha \rfloor}{m}$$

$$\text{Or, } \alpha \in [0; 1[\quad \text{alors on a: } \begin{aligned} 0 &\leq m\alpha < m \\ \Leftrightarrow 0 &\leq \lfloor m\alpha \rfloor < m \\ \Leftrightarrow 0 &\leq \frac{\lfloor m\alpha \rfloor}{m} < 1 \end{aligned} \quad \Bigg\} \div m > 0$$

On passe maintenant à la partie entière du tout:

$$\left\lfloor \frac{\lfloor mx \rfloor}{m} \right\rfloor = \left\lfloor k + \frac{\lfloor m\alpha \rfloor}{m} \right\rfloor = k + 0 = k = \lfloor x \rfloor$$