TD 6 exo 6

1) Pour $x \in \mathbb{R}$, on a :

$\sin^2(2x+\frac{\pi}{6}) = \cos^2(x+\frac{\pi}{3}) \iff \sin(2x+\frac{\pi}{6}) = \cos(x+\frac{\pi}{3})$ ou $\sin(2x+\frac{\pi}{6}) = -\cos(x+\frac{\pi}{3})$

$\iff \sin(2x+\frac{\pi}{6}) = \sin(\frac{\pi}{2}-x-\frac{\pi}{3})$ ou $\sin(2x+\frac{\pi}{6}) = \sin(x+\frac{\pi}{3}-\frac{\pi}{2})$ (*)

$\iff \exists k \in \mathbb{Z} : 2x+\frac{\pi}{6} = \frac{\pi}{6}-x+2k\pi$ ou $2x+\frac{\pi}{6} = \pi-(\frac{\pi}{6}-x)+2k\pi$

ou $2x+\frac{\pi}{6} = x-\frac{\pi}{6}+2k\pi$ ou $2x+\frac{\pi}{6} = \pi-(x-\frac{\pi}{6})+2k\pi$

$\iff \exists k \in \mathbb{Z} : 3x = 2k\pi$ ou $x = \frac{2\pi}{3}+2k\pi$

ou $x = -\frac{\pi}{3}+2k\pi$ ou $3x = \pi+2k\pi$

d'où l'ensemble des solutions $\mathcal{S}_1 = \{\frac{2k\pi}{3}, \frac{2\pi}{3}+2k\pi, -\frac{\pi}{3}+2k\pi, \frac{\pi}{3}+\frac{2k\pi}{3}, k \in \mathbb{Z}\}$.

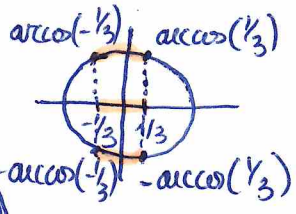
(*) : car $\cos(a) = \sin(\frac{\pi}{2}-a)$ et $-\cos(a) = \sin(a-\frac{\pi}{2})$

2) Pour $x \in [0, 2\pi[$ on a :

$3|\cos(x)| < 1 \iff |\cos(x)| < \frac{1}{3} \iff -\frac{1}{3} < \cos(x) < \frac{1}{3}$

$\iff \left(\arccos(\frac{1}{3}) < x < \arccos(-\frac{1}{3})\right.$

$\left.\text{ou } 2\pi-\arccos(-\frac{1}{3}) < x < 2\pi-\arccos(\frac{1}{3})\right)$

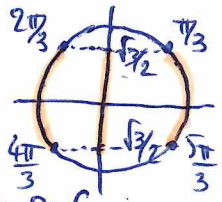


d'où l'ensemble des solutions $\mathcal{S}_2 =]\arccos(\frac{1}{3}), \arccos(-\frac{1}{3})[\cup]2\pi-\arccos(-\frac{1}{3}), 2\pi-\arccos(\frac{1}{3})[$.

3) Pour $x \in [0, 2\pi[$ on a :

$\sin^2(x) \leq \frac{3}{4} \iff -\sqrt{\frac{3}{4}} \leq \sin(x) \leq \sqrt{\frac{3}{4}} \iff -\frac{\sqrt{3}}{2} \leq \sin(x) \leq \frac{\sqrt{3}}{2}$

$\iff \left(0 \leq x \leq \frac{\pi}{3} \text{ ou } \frac{2\pi}{3} \leq x \leq \frac{4\pi}{3} \text{ ou } \frac{5\pi}{3} \leq x < 2\pi\right)$



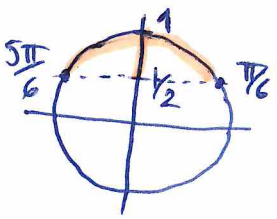
d'où l'ensemble des solutions $\mathcal{S}_3 = [0, \frac{\pi}{3}] \cup [\frac{2\pi}{3}, \frac{4\pi}{3}] \cup [\frac{5\pi}{3}, 2\pi[$.

4) Pour $x \in [0, 2\pi[$ on a :

$2\sin^2(x) - 3\sin(x) + 1 < 0 \iff \begin{cases} y = \sin(x) \\ 2y^2 - 3y + 1 < 0 \end{cases}$ (*).

Le discriminant de $2X^2 - 3X + 1$ est $(-3)^2 - 4 \times 2 \times 1 = 1$ donc ses racines sont $\frac{3+\sqrt{1}}{2\times 2} = 1$ et $\frac{3-\sqrt{1}}{2\times 2} = \frac{1}{2}$. Comme de plus son coefficient dominant est $2 > 0$, on a :

$(*) \iff \begin{cases} y = \sin(x) \\ \frac{1}{2} < y < 1 \end{cases} \iff \frac{1}{2} < \sin(x) < 1$



$\iff \frac{\pi}{6} < x < \frac{5\pi}{6}$

d'où l'ensemble des solutions $\mathcal{S}_4 =]\frac{\pi}{6}, \frac{5\pi}{6}[$.