NOM :

PRENOM :

Note sur 5 :

Question 1 (/1pt).
Calculer la forme algébrique de $z_1 = \dfrac{1}{(i-3)^2}$.

$$z_1 = \frac{1}{i^2-6i+9} = \frac{1}{8-6i}$$
$$= \frac{8+6i}{(8-6i)(8+6i)}$$
$$= \frac{8+6i}{8^2+6^2}$$
$$= \frac{8+6i}{100}$$
$$= \boxed{\frac{2}{25}+\frac{3}{50}i}$$

Question 2 (/1pt).
Calculer la forme exponentielle de $z_2 = 3-3i$.

$$|z_2| = |3-3i| = 3|1-i| = 3\sqrt{1+1} = 3\sqrt{2}$$
puis
$$z_2 = 3-3i = 3\sqrt{2}\left(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right)$$
$$= 3\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$$
$$= \boxed{3\sqrt{2}e^{-i\frac{\pi}{4}}}$$

Question 3 (/3pts).
Linéariser, pour $\theta \in \mathbb{R}$ $\cos^2(\theta)\sin(\theta)$.

$$\cos^2(\theta)\sin(\theta) = \left(\frac{e^{i\theta}+e^{-i\theta}}{2}\right)^2\left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right)$$
$$= \frac{1}{8i}(e^{i\theta}+e^{-i\theta})^2(e^{i\theta}-e^{-i\theta})$$
$$= \frac{1}{8i}(e^{2i\theta}+2+e^{-2i\theta})(e^{i\theta}-e^{-i\theta})$$
$$= \frac{1}{8i}(e^{3i\theta}+2e^{i\theta}+e^{-i\theta}-e^{i\theta}-2e^{-i\theta}-e^{-3i\theta})$$
$$= \frac{1}{8i}(e^{3i\theta}-e^{-3i\theta}+e^{i\theta}-e^{-i\theta})$$
$$= \frac{1}{8i}(2i\sin(3\theta)+2i\sin(\theta))$$
$$= \boxed{\frac{\sin(3\theta)+\sin(\theta)}{4}}$$

NOM :

PRENOM :

Note sur 5 :

Question 1 (/1pt).
Calculer la forme algébrique de $z_1 = \dfrac{1}{(i-2)^2}$.

$$z_1 = \frac{1}{i^2-4i+4} = \frac{1}{3-4i}$$
$$= \frac{3+4i}{(3-4i)(3+4i)}$$
$$= \frac{3+4i}{9+16}$$
$$= \boxed{\frac{3}{25}+\frac{4}{25}i}$$

Question 2 (/1pt).
Calculer la forme exponentielle de $z_2 = -2+2i$.

$$|z_2| = |-2+2i| = 2|-1+i| = 2\sqrt{1+1} = 2\sqrt{2}$$
$$z_2 = 2\sqrt{2}\left(-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)$$
$$= 2\sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)\right)$$
$$= \boxed{2\sqrt{2}e^{\frac{3i\pi}{4}}}$$

Question 3 (/3pts).
Linéariser, pour $\theta \in \mathbb{R}$ $\sin^2(\theta)\cos(\theta)$.

$$\sin^2(\theta)\cos(\theta) = \left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right)^2 \times \left(\frac{e^{i\theta}+e^{-i\theta}}{2}\right)$$
$$= \frac{1}{8i^2}\left(e^{i\theta}-e^{-i\theta}\right)^2\left(e^{i\theta}+e^{-i\theta}\right)$$
$$= -\frac{1}{8}\left(e^{2i\theta}-2+e^{-2i\theta}\right)\left(e^{i\theta}+e^{-i\theta}\right)$$
$$= -\frac{1}{8}\left(e^{3i\theta}-2e^{i\theta}+e^{-i\theta}+e^{i\theta}-2e^{-i\theta}+e^{-3i\theta}\right)$$
$$= -\frac{1}{8}\left(e^{3i\theta}+e^{-3i\theta}-e^{i\theta}-e^{-i\theta}\right)$$
$$= -\frac{1}{8}\left(2\cos(3\theta)-2\cos(\theta)\right)$$
$$= \boxed{\frac{\cos(\theta)-\cos(3\theta)}{4}}$$