TD 7

exo 19 :

$$C_n + iS_n = \sum_{k=0}^{n} \binom{n}{k} (\cos(kx) + i\sin(kx)) = \sum_{k=0}^{n} \binom{n}{k} e^{ikx} = \sum_{k=0}^{n} \binom{n}{k} (e^{ix})^k 1^{n-k}$$

$$= (e^{ix} + 1)^n = \left(e^{i\frac{x}{2}} \left(e^{i\frac{x}{2}} + e^{-i\frac{x}{2}} \right) \right)^n = e^{i\frac{nx}{2}} \left(e^{i\frac{x}{2}} + e^{-i\frac{x}{2}} \right)^n$$

$$= e^{i\frac{nx}{2}} \left(2\cos\left(\frac{x}{2}\right) \right)^n = 2^n \cos^n\left(\frac{x}{2}\right) \left(\cos\left(\frac{nx}{2}\right) + i\sin\left(\frac{nx}{2}\right) \right)$$

Donc $C_n = \mathrm{Re}(C_n + iS_n) = 2^n \cos^n\left(\frac{x}{2}\right) \cos\left(\frac{nx}{2}\right)$

et $S_n = \mathrm{Im}(C_n + iS_n) = 2^n \cos^n\left(\frac{x}{2}\right) \sin\left(\frac{nx}{2}\right)$