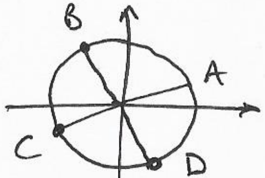
Outils mathématiques

I] $\cos(\overbrace{\omega t - kx + \tfrac{\pi}{2}}^{B}) = -\sin(\overbrace{\omega t - kx}^{A})$

$\cos(\underbrace{\omega t - kx + \pi}_{C}) = -\cos(\omega t - kx)$

$\sin(\underbrace{\omega t - kx}_{A}) = -\cos(\underbrace{\omega t - kx + \tfrac{\pi}{2}}_{B}) = \cos(\omega t - kx - \tfrac{\pi}{2})$



IV] a) $\frac{1}{2} - \frac{3}{4} = \frac{2-3}{4} = -\frac{1}{4}$

b) $\frac{\frac{1}{2}}{-\frac{3}{4}} = \frac{1}{2}\left(-\frac{4}{3}\right) = -\frac{2}{3}$

c) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{6+4+3}{12} = 1$

d) $\frac{1}{OA'} - \frac{1}{OA} = \frac{OA - OA'}{OA'\,OA}$

e) $\frac{1}{x+d} - \frac{1}{d} = \frac{1}{f'} \Leftrightarrow$ équivalent (et pas = !) $\frac{1}{x+d} = \frac{1}{f'} + \frac{1}{d} = \frac{f'+d}{f'd}$

(En inversant $\Leftrightarrow$) $x+d = \frac{f'd}{f'+d} \Leftrightarrow x = \frac{f'd}{f'+d} - d$

$= \frac{f'd - f'd - d^2}{f'+d}$

$\Leftrightarrow \boxed{x = \frac{-d^2}{f'+d}}$

f) On a $c_s(t_s - t_0) = d = c_p(t_p - t_0)$ (permet d'éliminer d)

donc $t_0(-c_s + c_p) = c_p t_p - c_s t_s \Rightarrow \boxed{t_0 = \frac{c_p t_p - c_s t_s}{c_p - c_s}}$

D'où $d = c_s(t_s - t_0) = c_s\left(\frac{t_s c_p - \cancel{c_s t_s} - c_p t_p + \cancel{c_s t_s}}{c_p - c_s}\right)$

Soit $\boxed{d = \frac{c_s c_p (t_s - t_p)}{c_p - c_s}}$

g) On a $\frac{f_1}{f_2} = \frac{c+v}{c-v}$ (permet d'éliminer f_0)

$\Leftrightarrow f_1(c-v) = f_2(c+v) \Leftrightarrow v(f_1+f_2) = c(f_1 - f_2)$

$\Rightarrow \boxed{v = \frac{c(f_1 - f_2)}{f_1 + f_2}}$

Alors $f_0 = f_2\left(\frac{c+v}{c}\right) = f_2\left(1+\frac{v}{c}\right) = f_2\left(1 + \frac{f_1 - f_2}{f_1+f_2}\right) = f_2 \frac{2f_1}{f_1+f_2}$

$\Rightarrow \boxed{f_0 = \frac{2 f_1 f_2}{f_1 + f_2}}$

V] 1) (E) $\Leftrightarrow \underbrace{2}_{a}x^2 + \underbrace{d}_{b}x + \underbrace{d^2 - 1}_{c} = 0$

Donc $\Delta = d^2 - 8(d^2 - 1) = -7d^2 + 8$; $\Delta \geq 0$ si $d^2 < \frac{8}{7}$

soit si $|d| < \sqrt{\frac{8}{7}}$. Alors $x_{1/2} = -\frac{d}{4} \pm \frac{\sqrt{8 - 7d^2}}{4}$

2) $\frac{1}{D-x} + \frac{1}{x} = \frac{1}{f'} \Leftrightarrow \frac{\cancel{x} + D - \cancel{x}}{x(D-x)} = \frac{1}{f'} \Leftrightarrow Df' = x(D-x)$

$\Leftrightarrow x^2 - Dx + Df' = 0$, de $\Delta = D^2 - 4Df'$. A des solutions si $D \geq 4f'$, et alors $x_{1/2} = \frac{D \pm \sqrt{\Delta}}{2}$

VI] 1) $f(0) = f_0 \Rightarrow f(t) = \frac{f(0)}{2}$ si $e^{-t/\tau} = \frac{1}{2} \Leftrightarrow -\frac{t}{\tau} = \ln\frac{1}{2} = -\ln 2$

Soit $\boxed{t = \tau \ln 2}$

2) De même, on a $1 - e^{-t/\tau} = 0{,}99 \Leftrightarrow e^{-t/\tau} = 0{,}01$

$\Leftrightarrow -\frac{t}{\tau} = \ln 0{,}01 = -\ln 100 \Rightarrow t = \tau \ln 100 \simeq 4{,}6\,\tau$

VII] On trouve : 1] 10^{-1} 2] 10^{-25} 3] 10^{2} 4] x^{-3a}

5] x^{2-4a} 6] $x^{3/2}$

IX] On trouve (à 1 cste près) : 1] $F(t) = \dfrac{\sin \omega t}{\omega}$ 2] $F(t) = -\dfrac{\cos \omega t}{\omega}$

3] $F(t) = \dfrac{1}{a} e^{at}$ 4] $F(t) = -\tau e^{-t/\tau}$ 5] $F(t) = \dfrac{\sin(\omega t + \varphi)}{\omega}$

6] $F(t) = \dfrac{\sin(4\omega t + \varphi)}{4\omega}$ 7] $F(t) = 2\dfrac{t^4}{4} + 6\dfrac{t^3}{3} + \dfrac{7t^2}{2} + t$

8] $$\int_0^\infty e^{-t/\tau} dt = \left[-\tau e^{-t/\tau}\right]_0^\infty = (-\tau e^{-\infty}) - (-\tau e^{-0}) = \tau$$

9] $$\frac{1}{T}\int_0^T s_0 \cos(\omega t + \varphi) dt = \frac{s_0}{T}\left[\frac{\sin(\omega t + \varphi)}{\omega}\right]_0^T = \frac{s_0}{T\omega}\left(\sin(\underset{2\pi}{\underset{\shortparallel}{\omega T}} + \varphi) - \sin(\varphi)\right)$$
$$= 0$$

$\left(\text{car } T = \dfrac{2\pi}{\omega}\right)$

10] $$\frac{1}{T}\int_0^T s_0 \sin(\omega t + \varphi) dt = 0$$ (même méthode)

11] $$\frac{1}{T}\int_0^T s_0^2 \cos^2(\omega t + \varphi) dt = \frac{1}{T}\int_0^T s_0^2 \left[\frac{1 + \cos(2\omega t + 2\varphi)}{2}\right] dt = A$$

$\left(\text{comme } \cos 2x = 2\cos^2 x - 1,\ \cos^2 x = \dfrac{\cos 2x + 1}{2}\right)$

$$A = \frac{S_0^2}{2T}\left[\int_0^T dt + \int_0^T \cos(2\omega t + 2\varphi) dt\right]$$
$$= \frac{S_0^2}{2T}\left[T + \underbrace{\left[\frac{\sin(2\omega t + 2\varphi)}{2\omega}\right]_0^T}_{0\ \left(= \sin(\underbrace{2\omega T}_{4\pi} + 2\varphi) - \sin(2\varphi)\right)}\right] = \frac{S_0^2}{2}$$

12] De même, on trouve
$$\frac{1}{T}\int_0^T s_0^2 \sin^2(\omega t + \varphi) dt = \frac{S_0^2}{2}$$