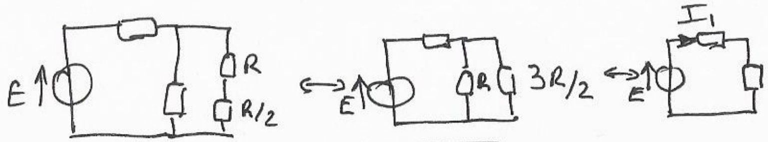
Corrigé DS

Problème 1

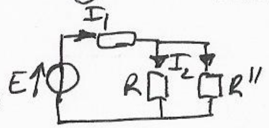
A) 1) On redessine le circuit en associant les résistances ; on obtient successivement



$\frac{3R^2}{\frac{3R}{2}+R} = \frac{3R}{5}$

D'où $R' = R + \frac{3R}{5} = \boxed{\frac{8R}{5} = R'}$ (2) Alors $\boxed{I_1 = \frac{E}{R'} = \frac{5E}{8R}}$ (Pouillet) (1)

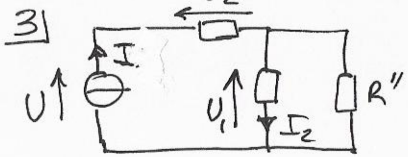
2) Déjà fait… $\boxed{R'' = \frac{3R}{2}}$ (1)



Alors $I_2 = \frac{R''}{R''+R} I_1$ (div courant)

Soit $I_2 = \frac{3R}{2\left(\frac{3R}{2}+R\right)} \frac{5E}{8R} = \boxed{\frac{3E}{8R} = I_2}$ (2)

3)



Alors $\boxed{I = I_0}$ (géné courant)

$\boxed{I_2 = \frac{3}{5} I_0}$ (même diviseur de courant) (1)

Alors (Loi des Mailles) : $U = U_1 + U_2 = R\left(I_0 + \frac{3}{5} I_0\right) = \boxed{\frac{8}{5} R I_0 = U}$ (1)

B) 4) On a 1 diviseur de tension avec $R /\!/ R_d$:

$$\boxed{V_1 = \frac{R/\!/R_d}{R_1+R_g+R/\!/R_d} E = \frac{\frac{RR_d}{R+R_d}}{R_1+R_g+\frac{RR_d}{R+R_d}} E = \frac{E R R_d}{RR_d + (R+R_d)(R_1+R_g)}}$$ (2)

5) Si $R_d \gg R$, $\frac{RR_d}{R+R_d} \simeq \frac{RR_d}{R_d} \simeq R$, Alors V_1 indep de R_d. (2)

6) Alors $\boxed{V_1 = \frac{R\ E}{R_1+R_g+R}}$ (on a remplacé $R/\!/R_d$ par R) (1)

7) On isole R : $V_1(R_1+R_g+R) = ER \Rightarrow \boxed{R = \frac{V_1(R_1+R_g)}{E - V_1}}$ (1)

AN : $\boxed{R = \frac{3(700)}{10-3} = 300\,\Omega}$ (2) Alors $\boxed{T = \frac{1}{\alpha}\left(\frac{R}{A} - 1\right) = 400\,°C}$ (2)

8) On a

Alors $P_R = \frac{U_R^2}{R} = \frac{1}{R}\left(\frac{R^2 E^2}{(R_1+R_2+R)^2}\right)$

$\boxed{P_R = \frac{R E^2}{(R_1+R_g+R)^2}}$ (1) (2)

9) Alors $\boxed{P_R = \frac{A(1+\alpha T)\, E^2}{(R_1+R_g+A(1+\alpha T))^2}}$ (1)

10) En posant $r = R_1 + R_g$, $P_R = \frac{RE^2}{(r+R)^2}$. P_R est max si

$\frac{dP}{dR} = 0$ (1) $\Leftrightarrow E^2\left(\frac{(r+R)^2 - 2(r+R)R}{(r+R)^4}\right) = 0$, soit si $r+R-2R=0$

$\Rightarrow P_R$ max pour $\boxed{R = r = R_1 + R_g = 700\,\Omega}$. (2)

Or on avec $R = A(1+\alpha T) \Rightarrow \boxed{T = \frac{1}{\alpha}\left(\frac{R}{A} - 1\right)}$ (1)

AN : $\boxed{T = \frac{1}{5\,10^{-3}}\left(\frac{700}{100} - 1\right) = \frac{6\,10^3}{5} = 1200\,K}$. (2)

11) $R = A(1+\alpha T) \Rightarrow \begin{cases} A \text{ en } \Omega \\ \alpha \text{ en } °C^{-1} \text{ (ou } K^{-1}) \end{cases}$ (2)

12) On résoud le système : $\begin{cases} R_1 = A(1+\alpha T_1) & (1) \\ R_2 = A(1+\alpha T_2) & (2) \end{cases}$

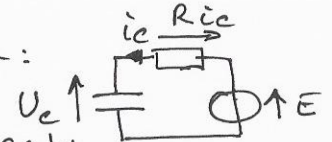
On élimine A en faisant (1)/(2) :

$\frac{R_1}{R_2} = \frac{1+\alpha T_1}{1+\alpha T_2} \Leftrightarrow R_1 + \alpha R_1 T_2 = R_2 + \alpha R_2 T_1 \Rightarrow \boxed{\alpha = \frac{R_2 - R_1}{R_1 T_2 - R_2 T_1}}$

AN : $\boxed{\alpha = \frac{300-140}{140 \times 100 - 300 \times 20} = \frac{160}{8000} = 2\,10^{-2}\,°C^{-1}}$

Alors $\boxed{A = \frac{R_1}{1+\alpha T_1} \underset{AN}{=} \frac{140}{1+2\,10^{-2} \times 20} = 100\,\Omega}$ (4)

Problème 2 | 1] Schéma :



Ldm : $E = U_c + Ri_c = U_c + RC\frac{dU_c}{dt}$

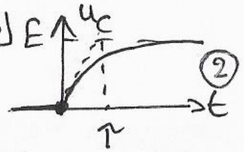
Soit (1) $\boxed{\frac{dU_c}{dt} + \frac{U_c}{\tau} = \frac{u_{c\infty}}{\tau}}$ avec $\boxed{\begin{cases}\tau = RC \\ U_{c\infty} = E\end{cases}}$ (3)

2] (1) a pour SG $u_c = \underbrace{Ae^{-t/\tau}}_{SH} + \underbrace{E}_{SP}$

à $t=0^-$, $U_c(0^-) = 0$ (énoncé). Comme Ucondensateur continue,

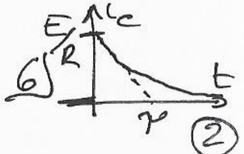
$U_c(0^+) = 0$. On remplace dans la SG : $0 = A + E \Rightarrow A = -E$

Finalement $\boxed{u_c = E(1-e^{-t/\tau})}$ (3)

3] 

4] On a $\boxed{\mathcal{E}_c = \frac{1}{2}CE^2}$ (1)

5] $i_c = C\frac{dU_c}{dt} = \frac{CE}{\tau}e^{-t/\tau} = \boxed{\frac{E}{R}e^{-t/\tau} = i_c}$ $(\tau = RC)$ (2)

6] 

(2)

7] L'énergie fournie par E vaut $\mathcal{E}_g = \int_0^\infty P_g\,dt = \int_0^\infty E.i\,dt$

Soit $\mathcal{E}_g = \int_0^R \frac{E^2}{R}e^{-t/\tau}dt = \frac{E^2}{R}\left[-\tau.e^{-t/\tau}\right]_0^\infty = \frac{E^2\tau}{R} = \boxed{CE^2 = \mathcal{E}_g}$ (3)

8] D'où $\boxed{\eta = \frac{\frac{1}{2}CE^2}{CE^2} = \frac{1}{2}}$ ne change pas si R change. — (2)

9] Même raisonnement, en changeant E en E/2 (τ inchangé)

$\Rightarrow \boxed{U_c = \frac{E}{2}(1-e^{-t/\tau})}$ (2)

10] à t_1, $U_c = 0{,}99\frac{E}{2} \Rightarrow 1-e^{-t/\tau} = 0{,}99 \Leftrightarrow e^{-t/\tau} = 0{,}01$

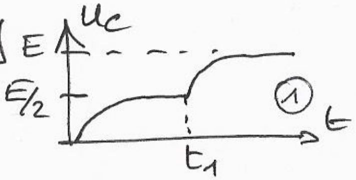
$\Leftrightarrow t_1 = -\tau \ln 0{,}01 = \tau\ln(100) = 5\tau \Rightarrow \boxed{t_1 = 5RC}$ (2)

11] Schéma identique à q1 ⇒ m̂ équation, m̂ SG.

La CI seule change. A $t' = t - t_1 = 0$, $U_c = \frac{E}{2}$. (nouveau temps, on change d'origine)

Alors $U_c(t'=0^+) = \frac{E}{2} \overset{SG}{=} A\underbrace{e^{-t'/\tau}}_{1} + E \Rightarrow A = -\frac{E}{2}$ (3)

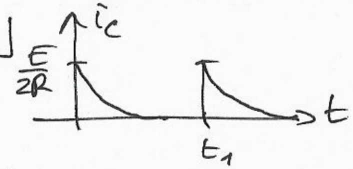
Finalement $u_c(t') = E\left(1 - \frac{e^{-t'/\tau}}{2}\right) \Leftrightarrow \boxed{U_c(t) = E\left(1 - \frac{1}{2}e^{-\frac{t-t_1}{\tau}}\right)}$

12] 

13] Avec $i_c = C\frac{dU_c}{dt}$, il vient :

$t < t_1$: $i_{c_1} = \frac{E}{2R}e^{-t/\tau}$

$t > t_1$: $i_{c_2} = \frac{E}{2R}e^{-\frac{t-t_1}{\tau}}$ (2)

14] 

(1)

15] Les générateurs fournissent :

$$\mathcal{E} = \int_0^{t_1}\frac{E}{2}\times i_{c_1}\,dt + \int_{t_1}^{\infty}E\times i_{c_2}\,dt$$

$$= \frac{E^2}{4R}\int_0^{t_1}e^{-t/\tau}dt + \frac{E^2}{2R}\int_{t_1}^{\infty}e^{-\frac{t-t_1}{\tau}}dt$$

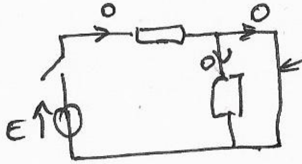
$$= \frac{E^2RC}{4R}\left[-e^{-t/\tau}\right]_0^{t_1} + \frac{E^2RC}{2R}\left[-e^{-\frac{t-t_1}{\tau}}\right]_{t_1}^{\infty}$$

$$= \frac{CE^2}{4}\left[1-\underbrace{e^{-5}}_{\simeq 0}\right] + \frac{CE^2}{2} \simeq \frac{3CE^2}{4}$$ (3)

16] On a $\boxed{\eta_2 = \frac{\frac{1}{2}CE^2}{\frac{3}{4}CE^2} = \frac{2}{3}}$ (1)

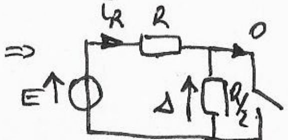
Problème 3

A 1) $t \leq 0$:



$s = 0$ (fil) $\Rightarrow \boxed{s(0^-) = 0}$ (1)

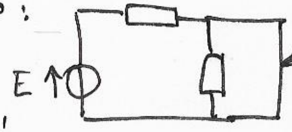
$t = 0^+$ i_{bobine} continue $\Rightarrow$



Donc div tension : $s(0^+) = E\frac{R/2}{R+R/2} = \boxed{\frac{E}{3} = s(0^+)}$ $\Rightarrow$ s discontinu (2)

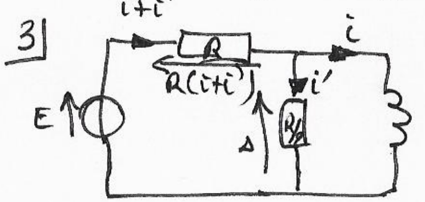
et $i_R = \frac{E}{R+R/2}$ (Pouillet) $\Leftrightarrow \boxed{i_R = \frac{2E}{3}}$ i_R discontinu (2)

2) $t \to \infty$:



$\boxed{s_\infty = 0 \ \text{(fil)}}$ (1)

3)



$s = L\frac{di}{dt} = \frac{R}{2}i' \Rightarrow i' = \frac{2s}{R}$

Loi des mailles à gauche : $E = R(i+i') + s$

$\Leftrightarrow E = Ri + 2s + s$. (1)

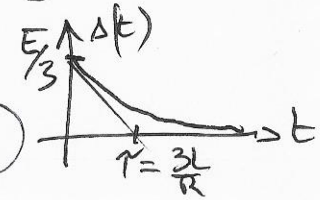
On dérive pour faire apparaître $\frac{di}{dt} = \frac{s}{L}$:

(1) $\Rightarrow 0 = R\frac{s}{L} + 3\frac{ds}{dt} \Leftrightarrow \boxed{\dot{s} + s\frac{R}{3L} = 0}$ (3)

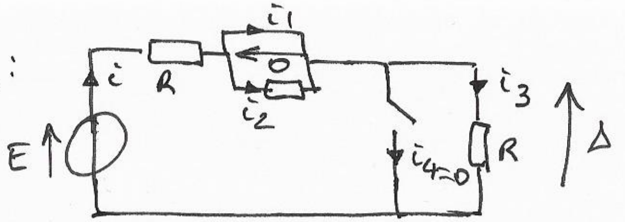
4) On a donc $s(t) = A e^{-t/\tau}$, avec $\boxed{\tau = \frac{3L}{R}}$

à $t = 0^+$, $s(0^+) \overset{CI}{=} \frac{E}{3} \overset{SG}{=} A \Rightarrow A = \frac{E}{3}$

D'où $\boxed{s(t) = \frac{E}{3} e^{-Rt/3L}}$ (4)



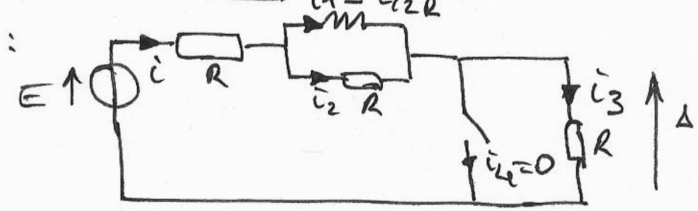
B 5) $t \leq 0$:



Tension nulle aux bornes des bobines $\Rightarrow Ri_2 = 0 \Rightarrow i_2 = 0$

Donc $\boxed{i = i_1 = i_3 = \frac{E}{2R}}$ (Pouillet) $\Rightarrow$.

$t = 0^+$:



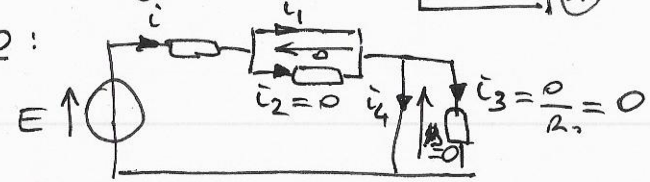
Par continuité de i dans une bobine : $\boxed{i_1(0^+) = \frac{E}{2R},\ i_4(0^+) = 0}$ (2)

Alors $i = i_3$ et $i_2 = i - \frac{E}{2R}$. (Loi des noeuds). La loi des mailles donne :

$$E = 2Ri + R\left(i - \frac{E}{2R}\right) = 3Ri - \frac{E}{2} \Rightarrow \boxed{i = \frac{E}{2R} = i_3}$$ (2)

Alors $\boxed{i_2 = 0}$ (1) et $s = Ri_3 = \boxed{\frac{E}{2} = s}$ (1)

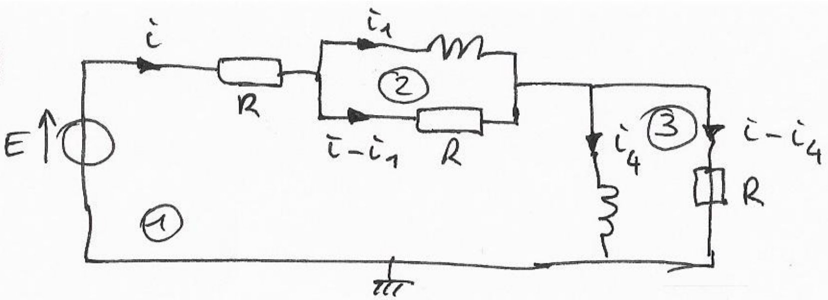
6) $t \to \infty$:



De même, $\boxed{i_3 = i_2 = 0}$ et $\boxed{i = i_1 = i_4 = \frac{E}{R}}$ (Pouillet)

$\Rightarrow \boxed{s = 0}$ (4)

7]



Maille (1) : $E = Ri + R(i-i_1) + L\frac{di_4}{dt}$ (1)

(2) : $R(i-i_1) = L\frac{di_1}{dt}$ (2)

(3) : $L\frac{di_4}{dt} = R(i-i_4)$ (3)

(3)

8] $(1)' \Rightarrow 2R\frac{di}{dt} - R\frac{di_1}{dt} + L\frac{d^2i_4}{dt^2} = 0 \Rightarrow \frac{di_1}{dt} = 2\frac{di}{dt} + \frac{L}{R}\frac{d^2i_4}{dt^2}$

$(2) \Rightarrow R(i-i_1) = 2L\frac{di}{dt} + \frac{L^2}{R}\frac{d^2i_4}{dt^2} \overset{(1)}{=} E - Ri - L\frac{di_4}{dt}$ (2')

$(3) \Rightarrow i = i_4 + \frac{L}{R}\frac{di_4}{dt}$

On remplace dans (2') :

$$E - R\left(i_4 + \frac{L}{R}\frac{di_4}{dt}\right) - L\frac{di_4}{dt} = 2L\frac{di_4}{dt} + 2\frac{L^2}{R}\frac{d^2i_4}{dt^2} + \frac{L^2}{R}\frac{d^2i_4}{dt^2}$$

soit $E = Ri_4 + 4L\frac{di_4}{dt} + \frac{3L^2}{R}\frac{d^2i_4}{dt^2}$

ou $\frac{d^2i_4}{dt^2} + \frac{4R}{3L}\frac{di_4}{dt} + \frac{1}{3}\left(\frac{R}{L}\right)^2 i_4 = \frac{ER}{3L^2}$ (3)

or $s = L\frac{di_4}{dt}$, en dérivant 3 il vient :

$$\frac{d^2s}{dt^2} + \frac{4R}{3L}\frac{ds}{dt} + \frac{1}{3}\left(\frac{R}{L}\right)^2 s = 0$$ (4)

on a donc par identification $\boxed{\omega_0 = \frac{R}{\sqrt{3}L}}$, $\frac{\omega_0}{Q} = 4\frac{R}{L} \Rightarrow Q = \frac{L\omega_0}{4R}$

soit $\boxed{Q = \frac{\sqrt{3}}{4}}$ (2)

$\boxed{1}$ **Exo TD**

1) $q = q'$ (2)

2) $q = \frac{Q}{2}\left(e^{-t/\tau} + 1\right)$ (2) $\dot{q} + \frac{q}{\tau} = \frac{Q}{RC}$ $\tau = \frac{RC}{2}$ (2)

3) $q_\infty = \frac{q}{2}$ (1)

4) $E_i = \frac{q^2}{2C}$, $E_f = \frac{q^2}{4C}$, $E_R = \frac{q^2}{4C}$ (3)