RSF

EC1 1/ a/ $u(t) = u_0 \cos\left(\omega t - \frac{\pi}{4}\right) \longleftrightarrow \underline{u}(t) = u_0 e^{j\omega t - \pi/4}$

$= \underbrace{u_0 e^{-j\pi/4}}_{\underline{u_0}} e^{j\omega t}$.

b/ $u(t) = -u_0 \cos(\omega t) = u_0 \cos(\omega t + \pi) \longleftrightarrow \underline{u}(t) = \underbrace{u_0 e^{-j\pi}}_{\underline{u_0}} e^{j\omega t}$

c/ $u(t) = u_0 \sin(\omega t) = u_0 \cos\left(\omega t - \frac{\pi}{2}\right) \longleftrightarrow \underline{u}(t) = \underbrace{u_0 e^{-j\pi/2}}_{\underline{u_0}} e^{j\omega t}$

2/ . Amplitude $U_0 = |\underline{u_0}| = u_0$

- phase à l'origine $\varphi = \arg(\underline{u_0}) = \pi$

3/ a/ - amplitude u_0

- phase à l'origine $\pi/3$ $\Rightarrow u(t) = u_0 \cos\left(\omega t + \frac{\pi}{3}\right)$

b/ $\underline{U_0} = u_0 j e^{j\pi/4} = u_0 e^{j(\pi/2 + \pi/4)} \Rightarrow u(t) = u_0 \cos\left(\omega t + \frac{3\pi}{4}\right)$

c/ $\underline{u_0} = -u_0 e^{j\pi/4} = u_0 e^{j(\pi + \frac{\pi}{4})} \Rightarrow u(t) = u_0 \cos\left(\omega t - \frac{3\pi}{4}\right)$

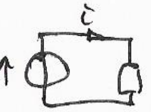
EC2 1/ $Z_{eq} = R + Z_C = R + \frac{1}{jC\omega} = \frac{1 + jRC\omega}{jC\omega}$

2/ $Z_{eq} = Z_L + Z_C = jL\omega + \frac{1}{jC\omega} = j\left(L\omega - \frac{1}{C\omega}\right)$

3/ $Z_{eq} = R + \frac{Z_L Z_C}{Z_L + Z_C} = R + \frac{Z_L}{1 + \frac{Z_L}{Z_C}} = R + \frac{jL\omega}{1 + (jL\omega)(jC\omega)}$

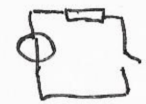
$\frac{1}{Z_C}$ est + simple... (evite des "étages" !..) $= R + \frac{jL\omega}{1 - LC\omega^2}$

EC3 . Schéma BF : (bobine = fil)



$\Rightarrow \underline{i} = \frac{\underline{e}}{R}$

. Schéma HF (bobine = —/ —)



$i = 0$

EC4 1/ $e(t) = E_0 \cos \omega t \longleftrightarrow \underline{e} = E_0 e^{j\omega t}$, amplitude complexe $\underline{E_0} = E_0$ (réelle ici)

2/ $i(t) = I_0 \cos(\omega t + \varphi) \longleftrightarrow \underline{i} = \underbrace{I_0 e^{j\varphi}}_{\underline{I_0}} e^{j\omega t}$ ampl cplx.

3/ Ldm ou Pouillet: $\underline{i} = \frac{\underline{e}}{R + jL\omega} \Leftrightarrow \underline{I_0} = \frac{E_0}{R + jL\omega}$ en amplitude complexe

4/ $I_0 = \frac{E_0}{\sqrt{R^2 + L^2\omega^2}}$, $\varphi = \arg \underline{I_0} = -\arctan\left(\frac{L\omega}{R}\right)$

Variante avec C : $\underline{i} = \frac{\underline{e}}{R + \frac{1}{jC\omega}} \Rightarrow I_0 = \frac{E_0}{\sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}}$,

$\underline{I_0} = \frac{E_0}{R - \frac{j}{C\omega}} \Rightarrow \varphi = -\arctan\left(-\frac{1}{RC\omega}\right) = \arctan\left(\frac{1}{RC\omega}\right)$

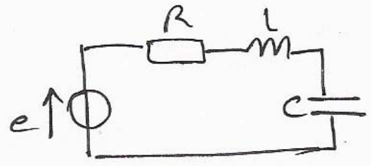
mettre j "au dessus" $\left(= \frac{\pi}{2} - \arctan(RC\omega) \ldots\right)$

EC5 Ldm complexe : $\underline{e} = (R + jL\omega)\underline{i}$.

Comme $j\omega \leftrightarrow$ dérivée, on retrouve l'équadif

$e(t) = Ri + L\frac{di}{dt}$

$\Rightarrow$ Si vous n'arrivez pas à établir l'eqdiff en transitoire "classiquement", passez en complexes !



- $\underline{e} = \underline{Z}\,\underline{i}$ avec $\underline{Z} = R + j\left(L\omega - \frac{1}{C\omega}\right)$

$\Leftrightarrow \underline{i} = \dfrac{\underline{e}}{R + j\left(L\omega - \frac{1}{C\omega}\right)} \Leftrightarrow \underline{I} = \dfrac{E/R}{1 + j\left(\frac{L\omega}{R} - \frac{1}{RC\omega}\right)}$ ← Im

A Retenir : $RC\omega$ sans U, $L\omega/R$ sans U, $LC\omega^2$

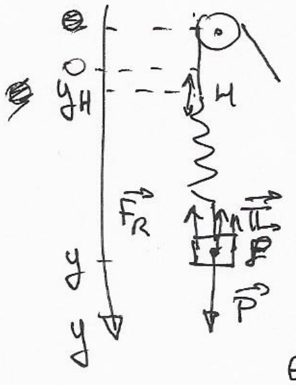
Identificat° : $\begin{cases} \frac{L\omega}{R} = Qx \\ \frac{1}{RC\omega} = \frac{Q}{x} \end{cases} \Rightarrow \begin{cases} Q^2 = \frac{1}{R^2}\frac{L}{C} \\ x^2 = LC\omega^2 \end{cases} \Rightarrow \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$

$x = \frac{\omega}{\omega_0} = LC\omega \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$

- Div tension $\underline{U_c} = \dfrac{Z_c\,\underline{e}}{Z_c + R + Z_L} = \dfrac{\underline{e}}{1 + \frac{R}{Z_c} + \frac{Z_L}{Z_c}} \Rightarrow \underline{U_c} = \dfrac{E}{1 + jRC\omega - LC\omega^2}$ — pas sympa

Identification : $x = \frac{\omega}{\omega_0} = \sqrt{LC}\,\omega \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$

$\frac{x}{Q} = \frac{\omega\sqrt{LC}}{Q} = RC\omega \Rightarrow \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$



- PFD pour R dans RTSG, proj sur Oy :

$m\ddot{y} = \underbrace{mg - \rho V g}_{m'g} - k(y - y_H) - \lambda\dot{y}$

$\Leftrightarrow \ddot{y} + \frac{\lambda}{m}\dot{y} + \frac{k}{m}y = \frac{m'}{m}g + \frac{kA}{m}\cos\omega t.$

$\Leftrightarrow \ddot{y} + \frac{\lambda}{m}\dot{y} + \frac{k}{m}(y - y_{eq}) = \frac{kA}{m}\cos\omega t$ avec $y_{eq} = \frac{m'g}{k}$

En posant $z = y - y_{eq}$, $\ddot{z} + \frac{\omega_0}{Q}\dot{z} + \omega_0^2 z = \omega_0^2 A\cos\omega t$,

avec $\boxed{\omega_0 = \sqrt{\frac{k}{m}}}$, et $\frac{\omega_0}{Q} = \frac{1}{Q}\sqrt{\frac{k}{m}} = \frac{\lambda}{m} \Rightarrow \boxed{Q = \frac{\sqrt{km}}{\lambda}}$

Soit en complexes : $\underline{Z}\left(-\omega^2 + j\omega\frac{\omega_0}{Q} + \omega_0^2\right) = \omega_0^2 A$

$\Leftrightarrow \underline{Z}\left(1 + j\frac{x}{Q} - x^2\right) = \underline{A}$ (en divisant par ω_0^2 posant $x = \frac{\omega}{\omega_0}$)

$\Leftrightarrow \boxed{\underline{Z} = \dfrac{A}{1 + j\frac{x}{Q} - x^2}}$

- Pour la vitesse, $v = \frac{dx}{dt} \Rightarrow \underline{\dot{Z}} = j\omega\underline{Z} = jx\underline{Z}\omega_0$ $\boxed{j\omega = jx\omega_0}$ ♡

$\Rightarrow \underline{\dot{Z}} = \dfrac{jx\omega_0 A}{1 + j\frac{x}{Q} - x^2} = \dfrac{\omega_0 A}{\frac{1}{jx} + \frac{1}{Q} - \frac{x^2}{jx}} = \dfrac{\omega_0 A}{j\left(-\frac{1}{x} + x\right) + \frac{1}{Q}}$

$\Leftrightarrow \boxed{\underline{\dot{Z}} = \dfrac{\omega_0 A Q}{1 + jQ\left(x - \frac{1}{x}\right)}}$

2) Resonance $\cancel{z}$, U_c

$$\underline{U_c} = \frac{U_0}{1 + j\frac{x}{Q} - x^2}$$

Module $\Rightarrow$ $U_c = \dfrac{U_0}{\sqrt{\frac{x^2}{Q^2} + (1-x^2)^2}} = \dfrac{U_0}{\sqrt{D(x)}}$

• amplitude

savoir refaire (astuce)

U_c max si $D(x)$ min $\Leftrightarrow \frac{dD}{dx} = 0 \Leftrightarrow \frac{2\cancel{x}}{Q^2} - 4\cancel{x}(1-x^2) = 0$

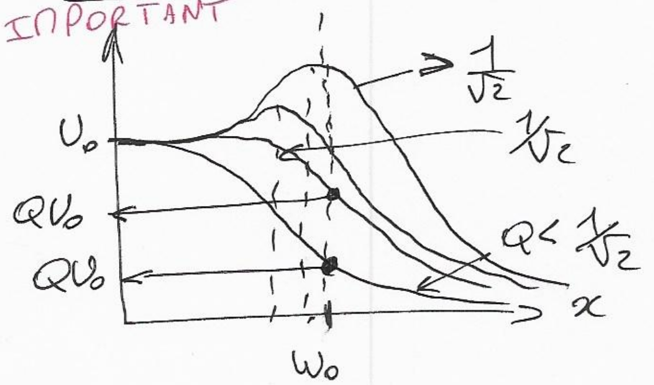
astuce !

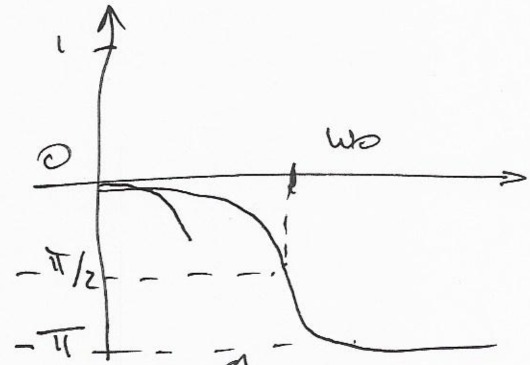
$\Leftrightarrow \frac{1}{2Q^2} - 1 + x^2 = 0 \Leftrightarrow x = \sqrt{1 - \frac{1}{2Q^2}}$, si $\boxed{Q > \frac{1}{\sqrt{2}}}$

$\Leftrightarrow \boxed{\omega_r = \omega_0\sqrt{1 - \frac{1}{2Q^2}}} < \omega_0$ (proche si Q gd)

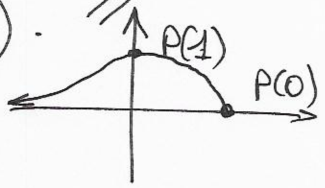
Rque : $\boxed{U(x=1) = QU_0}$ $\Rightarrow$ accès à Q

IMPORTANT





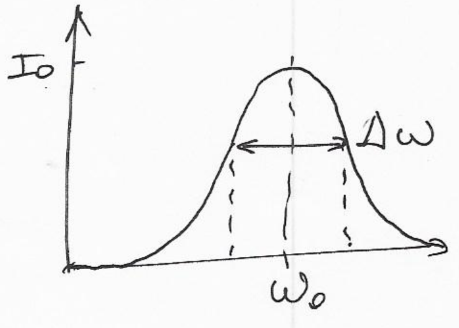
• phase : $\varphi = -\arg\left(\underbrace{(1-x^2) + j\frac{x}{Q}}_{\underline{P}(x)}\right)$.



3) Résonance i, v $\underline{I} = \dfrac{I_0}{1 + jQ\left(x - \frac{1}{x}\right)}$.

• amplitude $I = \dfrac{I_0}{\sqrt{1 + Q^2\underbrace{\left(x - \frac{1}{x}\right)^2}_{\text{au mini} = 0}}}$ $\Rightarrow I =$ ~~I~~ $I_{max} = I_0$ pour $x = 0$

$\Leftrightarrow \boxed{\omega_r = \omega_0}$



• Bande passante : on cherche $x_{1/2}$ tq

savoir refaire (astuce...)

$I(x) = \frac{I_0}{\sqrt{2}} \Leftrightarrow 1 + Q^2\left(x - \frac{1}{x}\right)^2 = 2$

$\Leftrightarrow \left(x - \frac{1}{x}\right) = \pm\frac{1}{Q}$

$\Leftrightarrow x^2 \pm \frac{x}{Q} - 1 = 0$: on prend les 2 racines positives de ces 2 equations $x_i = \frac{-b \pm \sqrt{\Delta}}{2a}$, soit $\begin{cases} x_1 = \frac{1}{2Q} + \frac{1}{2}\sqrt{\frac{1}{Q^2} + 4} \\ x_2 = -\frac{1}{2Q} + \frac{1}{2}\sqrt{\frac{1}{Q^2} + 4} \end{cases}$

$\Rightarrow \Delta x = \frac{1}{Q} \Leftrightarrow \boxed{\Delta\omega = \frac{\omega_0}{Q}}$ $\downarrow$ si $Q \nearrow$