Spirale logarithmique

2] On élimine $t = \frac{\theta}{\omega} \Rightarrow \boxed{r = b\, e^{-\theta/\omega\tau}}$

$$\begin{cases} r(\theta_0) = b\, e^{-\theta_0/\omega\tau} \\ r(\theta_0 + 2\pi) = b\, e^{-(\theta_0+2\pi)/\omega\tau} = r(\theta_0)\, e^{-2\pi/\omega\tau} \end{cases}$$

3] $$\begin{cases} \dot{r} = v_{\text{radiale}} = -\frac{b}{\tau} e^{-t/\tau} \\ r\dot{\theta} = v_{\text{orthoradiale}} = b\omega e^{-t/\tau} \end{cases}$$

$$a_{\text{radiale}} = \ddot{r} - r\dot{\theta}^2 = b\left(\frac{1}{\tau^2} - \omega^2\right) e^{-t/\tau}$$

$$a_{\text{orthoradiale}} = \underbrace{r\ddot{\theta}}_{0} + 2\dot{r}\dot{\theta} = -\frac{2b\omega}{\tau} e^{-t/\tau}$$

4] $$v = \sqrt{v_{\text{rad}}^2 + v_{\text{orth}}^2} = b\sqrt{\omega^2 + \frac{1}{\tau^2}}\, e^{-t/\tau}$$

$$a = \sqrt{a_{\text{rad}}^2 + a_{\text{orth}}^2} = b\sqrt{\left(\frac{1}{\tau^2} - \omega^2\right)^2 + \left(\frac{2b\omega}{\tau}\right)^2}\, e^{-t/\tau}$$

$$= b\sqrt{\frac{1}{\tau^4} - \frac{2\omega^2}{\tau^2} + \omega^4 + \frac{2b^2\omega^2}{\tau^2}}\, e^{-t/\tau}$$

$$= b\left(\frac{1}{\tau^2} + \omega^2\right) e^{-t/\tau}$$