RSF

EC1 1] a] $u(t) = u_0 \cos\left(\omega t - \frac{\pi}{4}\right) \leftrightarrow \underline{u}(t) = u_0 e^{j\omega t - \pi/4}$

$= \underbrace{u_0 e^{-j\pi/4}}_{\underline{u_0}} e^{j\omega t}$.

b] $u(t) = -u_0 \cos(\omega t) = u_0 \cos(\omega t + \pi) \leftrightarrow \underline{u}(t) = \underbrace{u_0 e^{-j\pi}}_{\underline{u_0}} e^{j\omega t}$

c] $u(t) = u_0 \sin(\omega t) = u_0 \cos\left(\omega t - \frac{\pi}{2}\right) \leftrightarrow \underline{u}(t) = \underbrace{u_0 e^{-j\pi/2}}_{\underline{u_0}} e^{j\omega t}$

2] - Amplitude $u_0 = |\underline{u_0}| = u_0$

- phase à l'origine $\varphi = \arg(\underline{u_0}) = \pi$

3] a] - amplitude u_0

- phase à l'origine $\pi/3$ $\Rightarrow u(t) = u_0 \cos\left(\omega t + \frac{\pi}{3}\right)$

b] $\underline{u_0} = u_0 j e^{j\pi/4} = u_0 e^{j(\pi/2 + \pi/4)} \Rightarrow u(t) = u_0 \cos\left(\omega t + \frac{3\pi}{4}\right)$

c] $\underline{u_0} = -u_0 e^{j\pi/4} = u_0 e^{j\left(\pi + \frac{\pi}{4}\right)} \Rightarrow u(t) = u_0 \cos\left(\omega t - \frac{3\pi}{4}\right)$

EC2 1] $Z_{eq} = R + Z_C = R + \frac{1}{jC\omega} = \frac{1 + jRC\omega}{jC\omega}$

2] $Z_{eq} = Z_L + Z_C = jL\omega + \frac{1}{jC\omega} = j\left(L\omega - \frac{1}{C\omega}\right)$

3] $Z_{eq} = R + \frac{Z_L Z_C}{Z_L + Z_C} = R + \frac{Z_L}{1 + \frac{Z_L}{Z_C}} = R + \frac{jL\omega}{1 + (jL\omega)(jC\omega)}$

$\frac{1}{Z_C}$ est + simple ... (évite des "étages" ..)

$= R + \frac{jL\omega}{1 - LC\omega^2}$

EC3 • Schéma BF : (bobine = fil) e ↑ — i — $\Rightarrow \underline{i} = \frac{\underline{e}}{R}$

• Schéma HF (bobine = —/—) $i = 0$

EC4 1] $e(t) = E_0 \cos \omega t \leftrightarrow \underline{e} = E_0 e^{j\omega t}$, amplitude complexe $\underline{E_0} = E_0$ (réelle ici)

2] $i(t) = I_0 \cos(\omega t + \varphi) \leftrightarrow \underline{i} = \underbrace{I_0 e^{j\varphi}}_{\underline{I_0}\ \text{ampl cplx}} e^{j\omega t}$

3] Ldm ou Pouillet : $\underline{i} = \frac{\underline{e}}{R + jL\omega} \Leftrightarrow \underline{I_0} = \frac{E_0}{R + jL\omega}$ en amplitude complexe

4] $I_0 = \frac{E_0}{\sqrt{R^2 + L^2\omega^2}}$, $\varphi = \arg \underline{I_0} = -\arctan\left(\frac{L\omega}{R}\right)$

Variante avec C : $\underline{i} = \frac{\underline{e}}{R + \frac{1}{jC\omega}} \Rightarrow I_0 = \frac{E_0}{\sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}}$,

$\underline{I_0} = \frac{E_0}{R - \frac{j}{C\omega}} \Rightarrow \varphi = -\arctan\left(-\frac{1}{RC\omega}\right) = \arctan\left(\frac{1}{RC\omega}\right)$

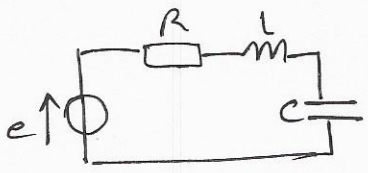
mettre $-j$ "au dessus" $\left(= \frac{\pi}{2} - \arctan(RC\omega) \ldots\right)$

EC5 Ldm complexe : $\underline{e} = (R + jL\omega)\underline{i}$.

Comme $j\omega \leftrightarrow$ dérivée, on retrouve l'équadiff

$e(t) = Ri + L\frac{di}{dt}$

⇒ Si vous n'arrivez pas à établir l'éq diff en transitoire "classiquement", passez en complexes !



• $\underline{e} = \underline{Z}\,\underline{i}$ avec $\underline{Z} = R + j\left(L\omega - \frac{1}{C\omega}\right)$

$\Leftrightarrow \underline{i} = \frac{\underline{e}}{R + j\left(L\omega - \frac{1}{C\omega}\right)} \Leftrightarrow \underline{I} = \frac{E/R}{1 + j\left(\frac{L\omega}{R} - \frac{1}{RC\omega}\right)}$ ← I_m

A Retenir : RCω sans U, Lω/R sans U, LCω² sans U

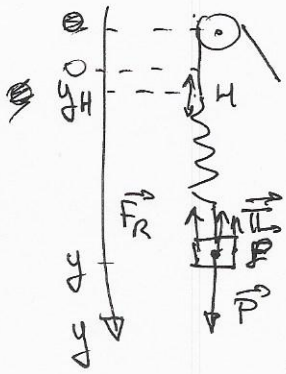
Identification : $\begin{cases} \frac{L\omega}{R} = Qx \\ \frac{1}{RC\omega} = \frac{Q}{x} \end{cases} \Rightarrow \begin{cases} Q^2 = \frac{1}{R^2}\frac{L}{C} \\ x^2 = LC\omega^2 \end{cases} \Rightarrow \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$

$x = \frac{\omega}{\omega_0} = LC\omega \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$

• Div tension $\underline{U_c} = \frac{Z_c\,\underline{e}}{Z_c + R + Z_L} = \frac{\underline{e}}{1 + \frac{R}{Z_c} + \frac{Z_L}{Z_c}}$ (pas sympa) $\Rightarrow \underline{U_c} = \frac{E}{1 + jRC\omega - LC\omega^2}$

Identification : $x = \frac{\omega}{\omega_0} = \sqrt{LC}\,\omega \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$

$\frac{x}{Q} = \frac{\omega\sqrt{LC}}{Q} = RC\omega \Rightarrow \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$



• PFD pour P dans RTSG, proj sur O_y :

$m\ddot{y} = \underbrace{mg - \rho V g}_{m'g} - k(y - y_H - l_0) - \lambda\dot{y}$

$\Leftrightarrow \ddot{y} + \frac{\lambda}{m}\dot{y} + \frac{k}{m}(y - l_0) = \frac{m'}{m}g + \frac{kA}{m}\cos\omega t$.

$\Leftrightarrow \ddot{y} + \frac{\lambda}{m}\dot{y} + \frac{k}{m}(y - y_{eq}) = \frac{kA}{m}\cos\omega t$ avec $y_{eq} = \frac{m'g}{k} + l_0$

En posant $z = y - y_{eq}$, $\ddot{z} + \frac{\omega_0}{Q}\dot{z} + \omega_0^2 z = \omega_0^2 A\cos\omega t$,

avec $\boxed{\omega_0 = \sqrt{\frac{k}{m}}}$, et $\frac{\omega_0}{Q} = \frac{1}{Q}\sqrt{\frac{k}{m}} = \frac{\lambda}{m} \Rightarrow \boxed{Q = \frac{\sqrt{km}}{\lambda}}$

Soit en complexes : $\underline{Z}\left(-\omega^2 + j\omega\frac{\omega_0}{Q} + \omega_0^2\right) = \omega_0^2 A$

$\Leftrightarrow \underline{Z}\left(1 + j\frac{x}{Q} - x^2\right) = H$ (en divisant par ω_0^2, posant $x = \frac{\omega}{\omega_0}$)

$\Leftrightarrow \boxed{\underline{Z} = \frac{A}{1 + j\frac{x}{Q} - x^2}}$

• Pour la vitesse, $v = \frac{dx}{dt} \Rightarrow \dot{\underline{z}} = j\omega\underline{z} = jx\underline{z}\omega_0$ $\quad \boxed{j\omega = jx\omega_0}$ ♡

$\Rightarrow \dot{\underline{Z}} = \frac{jx\omega_0 A}{1 + j\frac{x}{Q} - x^2} = \frac{\omega_0 A}{\frac{1}{jx} + \frac{1}{Q} - \frac{x^2}{jx}} = \frac{\omega_0 A}{j\left(-\frac{1}{x} + x\right) + \frac{1}{Q}}$

$\Leftrightarrow \boxed{\dot{\underline{Z}} = \frac{\omega_0 AQ}{1 + jQ\left(x - \frac{1}{x}\right)}}$

2) Résonance u_c

$$\underline{U_c} = \frac{U_0}{1 + j\frac{x}{Q} - x^2} \xrightarrow{\text{module}} U_c = \frac{U_0}{\sqrt{\frac{x^2}{Q^2} + (1-x^2)^2}} = \frac{U_0}{\sqrt{D(x)}}$$

• amplitude

Savoir refaire (astuce)

U_c max si $D(x)$ min $\Leftrightarrow \frac{dD}{dx} = 0 \Leftrightarrow \frac{2x}{Q^2} - 4x(1-x^2) = 0$

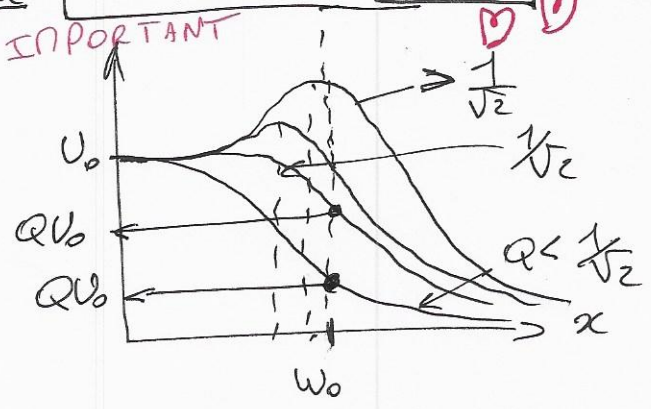
astuce!

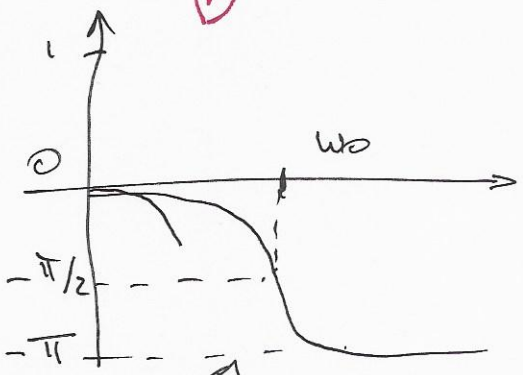
$$\Leftrightarrow \frac{1}{2Q^2} - 1 + x^2 = 0 \Leftrightarrow x = \sqrt{1 - \frac{1}{2Q^2}}, \text{ si } \boxed{Q > \frac{1}{\sqrt{2}}}$$

$$\Leftrightarrow \boxed{\omega_r = \omega_0 \sqrt{1 - \frac{1}{2Q^2}} < \omega_0} \text{ (proche si } Q \text{ gd)}$$

Rque: $\boxed{U(x=1) = Q U_0 \Rightarrow \text{accès à } Q}$

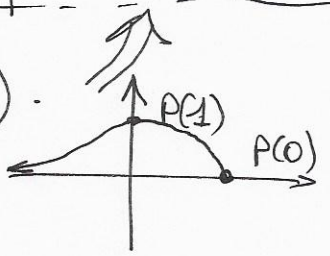
IMPORTANT





• phase : $\varphi = -\arg\left(\underbrace{(1-x^2) + j\frac{x}{Q}}_{\underline{P}(x)}\right)$.

comprendre!



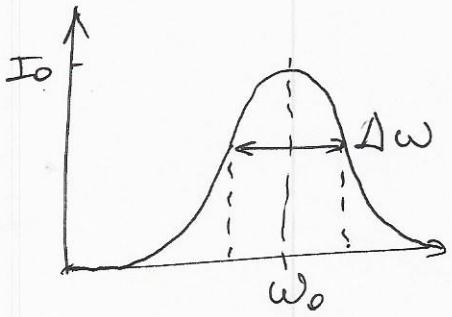
3) Résonance i, U

$$\underline{I} = \frac{I_0}{1 + jQ\left(x - \frac{1}{x}\right)}$$

• amplitude $I = \frac{I_0}{\sqrt{1 + Q^2\left(x - \frac{1}{x}\right)^2}} \Rightarrow I = I_{max} = I_0$ pour $x = 1$

$\Leftrightarrow \boxed{\omega_r = \omega_0}$

$\left(x - \frac{1}{x}\right)^2$ au mini $= 0$



• Bande passante : on cherche $x_{1/2}$ tq

$$I(x) = \frac{I_0}{\sqrt{2}} \Leftrightarrow 1 + Q^2\left(x - \frac{1}{x}\right)^2 = 2$$

$$\Leftrightarrow \left(x - \frac{1}{x}\right) = \pm\frac{1}{Q}$$

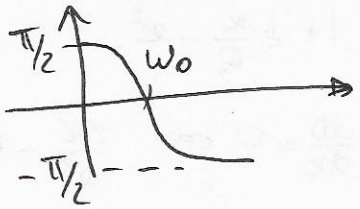
$\Leftrightarrow x^2 \pm \frac{x}{Q} - 1 = 0$: on prend les 2 racines positives de ces 2 équations $x_i = \frac{-b \pm \sqrt{\Delta}}{2a}$, soit

$$\begin{cases} x_1 = \frac{1}{2Q} + \frac{1}{2}\sqrt{\frac{1}{Q^2} + 4} \\ x_2 = -\frac{1}{2Q} + \frac{1}{2}\sqrt{\frac{1}{Q^2} + 4} \end{cases}$$

$\Rightarrow \Delta x = \frac{1}{Q} \Leftrightarrow \boxed{\Delta\omega = \frac{\omega_0}{Q}}$ ↓ si Q ↑

Savoir refaire (astuce)

• <u>phase :</u> s = dérivée de x ⇒ phase $+\pi/2$



ou se retrouve directement $\varphi = -\arctan\left[Q\left(x - \frac{1}{x}\right)\right]$