

TRIGONOMETRIE

I – Valeurs remarquables

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

II – Relation entre cos et sin

$$\forall x \in \mathbb{R}, \sin^2 x + \cos^2 x = 1$$

III – Angles associés

Une lecture efficace du cercle trigonométrique permet de retrouver les relations suivantes :

$$\cos\left(\frac{\pi}{2} + x\right) = -\sin x$$

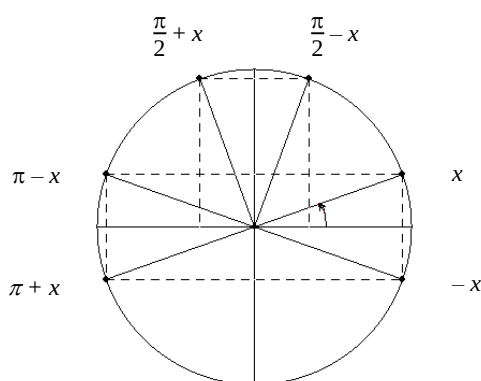
$$\sin\left(\frac{\pi}{2} + x\right) = \cos x$$

$$\cos(\pi - x) = -\cos x$$

$$\sin(\pi - x) = \sin x$$

$$\cos(\pi + x) = -\cos x$$

$$\sin(\pi + x) = -\sin x$$



$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos(-x) = \cos x$$

$$\sin(-x) = -\sin x$$

$$\cos(x + 2\pi) = \cos x$$

$$\sin(x + 2\pi) = \sin x$$

IV – Formules usuelles

Formules d'addition.

$$\forall (a, b) \in \mathbb{R}^2,$$

$$\begin{aligned} \cos(a + b) &= \cos a \cos b - \sin a \sin b \\ \cos(a - b) &= \cos a \cos b + \sin a \sin b \\ \sin(a + b) &= \sin a \cos b + \sin b \cos a \\ \sin(a - b) &= \sin a \cos b - \sin b \cos a \end{aligned}$$

Formules de duplication.

$$\forall a \in \mathbb{R},$$

$$\begin{aligned} \sin(2a) &= 2 \sin a \cos a \\ \cos(2a) &= \cos^2 a - \sin^2 a = 2 \cos^2 a - 1 = 1 - 2 \sin^2 a \end{aligned}$$

On en déduit :

$$\forall a \in \mathbb{R},$$

$$\begin{aligned} \cos^2 a &= \frac{1}{2} (1 + \cos(2a)) \\ \sin^2 a &= \frac{1}{2} (1 - \cos(2a)) \end{aligned}$$

V – Equations trigonométriques

$$\cos U = \cos V \Leftrightarrow (U = V + 2k\pi \text{ ou } U = -V + 2k\pi, k \in \mathbb{Z})$$

$$\sin U = \sin V \Leftrightarrow (U = V + 2k\pi \text{ ou } U = \pi - V + 2k\pi, k \in \mathbb{Z})$$

