

Développements limités usuels

À savoir par coeur!

Théorème .1 (Formule de Taylor-Young)

Soit f une fonction de classe \mathcal{C}^n sur I et $a \in I$, alors f admet un $DL_n(a)$:

$$f(x) \underset{x \rightarrow a}{=} f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2}f''(a) + \dots + \frac{(x-a)^n}{n!}f^{(n)}(a) + o((x-a)^n)$$
$$f(a+h) \underset{h \rightarrow 0}{=} f(a) + hf'(a) + \frac{h^2}{2}f''(a) + \dots + \frac{h^n}{n!}f^{(n)}(a) + o(h^n)$$

$$e^x \underset{x \rightarrow 0}{=} 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + o(x^n)$$

$$\sin(x) \underset{x \rightarrow 0}{=} x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$$

$$\cos(x) \underset{x \rightarrow 0}{=} 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$$

$$\operatorname{sh}(x) \underset{x \rightarrow 0}{=} x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$$

$$\operatorname{ch}(x) \underset{x \rightarrow 0}{=} 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$$

$$\frac{1}{1-x} \underset{x \rightarrow 0}{=} 1 + x + x^2 + \dots + x^n + o(x^n)$$

$$\ln(1-x) \underset{x \rightarrow 0}{=} -x - \frac{x^2}{2} - \frac{x^3}{3} - \dots - \frac{x^n}{n} + o(x^n)$$

$$\frac{1}{1+x} \underset{x \rightarrow 0}{=} 1 - x + x^2 - \dots + (-1)^n x^n + o(x^n)$$

$$\ln(1+x) \underset{x \rightarrow 0}{=} x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{n-1} \frac{x^n}{n} + o(x^n)$$

$$(1+x)^\alpha \underset{x \rightarrow 0}{=} 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \dots + \frac{\alpha(\alpha-1)\dots(\alpha-n+1)}{n!}x^n + o(x^n)$$

$$\arctan x \underset{x \rightarrow 0}{=} x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})$$

$$\tan x \underset{x \rightarrow 0}{=} x + \frac{x^3}{3} + \frac{2x^5}{15} + o(x^5)$$