TRIGONOMETRIE

I – Valeurs remarquables

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

II – Relations entre cos et sin

$\forall\, x \in \mathbb{R},\ \sin^2 x + \cos^2 x = 1$

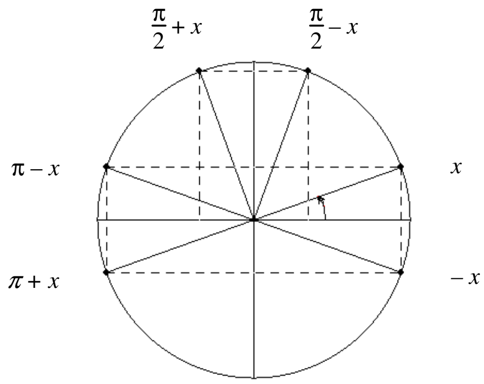
III – Angles associés

Une lecture efficace du cercle trigonométrique permet de retrouver les relations suivantes :

$$\cos\left(\frac{\pi}{2}+x\right) = -\sin x$$
$$\sin\left(\frac{\pi}{2}+x\right) = \cos x$$

$$\cos(\pi - x) = -\cos x$$
$$\sin(\pi - x) = \sin x$$

$$\cos(\pi + x) = -\cos x$$
$$\sin(\pi + x) = -\sin x$$



$$\cos\left(\frac{\pi}{2}-x\right) = \sin x$$
$$\sin\left(\frac{\pi}{2}-x\right) = \cos x$$

$$\cos(-x) = \cos x$$
$$\sin(-x) = -\sin x$$

IV – Formules usuelles

Formules d'addition.

$\forall\, (a, b) \in \mathbb{R}^2,$

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$
$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$
$$\sin(a + b) = \sin a \cos b + \sin b \cos a$$
$$\sin(a - b) = \sin a \cos b - \sin b \cos a$$

Formules de duplication.

$\forall\, a \in \mathbb{R},$

$$\sin(2a) = 2 \sin a \cos a$$
$$\cos(2a) = \cos^2 a - \sin^2 a = 2\cos^2 a - 1 = 1 - 2\sin^2 a$$

On en déduit :

$\forall\, a \in \mathbb{R},$

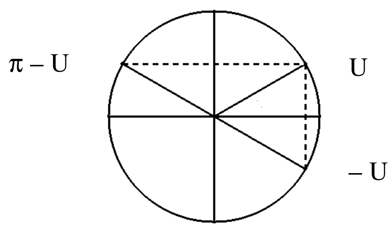
$$\cos^2 a = \frac{1}{2}(1 + \cos(2a))$$
$$\sin^2 a = \frac{1}{2}(1 - \cos(2a))$$

Formule de Moivre. $\forall\, \theta \in \mathbb{R}, \forall\, n \in \mathbb{N},\ (\cos\theta + i\sin\theta)^n = (e^{i\theta})^n = e^{in\theta} = \cos(n\theta) + i\sin(n\theta)$

V – Equations trigonométriques

$$\cos U = \cos V \iff (U \equiv V\ [2\pi]\ \text{ ou }\ U \equiv -V\ [2\pi])$$
$$\iff U = V + 2k\pi \text{ ou } U = -V + 2k\pi,\ k \in \mathbb{Z}$$

$$\sin U = \sin V \iff (U \equiv V\ [2\pi]\ \text{ ou }\ U \equiv \pi - V\ [2\pi])$$
$$\iff U = V + 2k\pi \quad \text{ou} \quad U = \pi - V + 2k\pi,\ k \in \mathbb{Z}$$



Cas particuliers :

$$\cos x = 0 \iff x \equiv \frac{\pi}{2}\ [\pi] \iff x = \frac{\pi}{2} + k\pi,\ \ k \in \mathbb{Z}$$

$$\sin x = 0 \iff x \equiv 0\ [\pi] \iff x = k\pi,\ \ k \in \mathbb{Z}$$