BANQUE PT 2006

EPREUVE DE PHYSIQUE A

PARTIE C : Etude d'un wattmètre électronique

1 - Puissance active

1.1) on passe en notation complexe : $\underline{v} = V_n e^{j\omega t}$

$\underline{i} = I_n e^{j(\omega t - \varphi)}$

on a $\underline{v} = \underline{Z}\,\underline{i} = (R + jX)\,\underline{i}$

$$\Rightarrow V_n e^{j\omega t} = (R+jX) I_n e^{j\omega t} e^{-j\varphi}$$

$$\Rightarrow I_n e^{-j\varphi} = \frac{V_n}{R+jX}$$

* $\boxed{I_n = \dfrac{V_n}{\sqrt{R^2+X^2}}}$

* $-\varphi = \arg\left(\dfrac{V_n}{R+jX}\right) = -\arg(R+jX)$

$$\Rightarrow \varphi = \arg(R+jX)$$

$$\Rightarrow \cos\varphi = \frac{R}{\sqrt{R^2+X^2}} \quad \text{et} \quad \sin\varphi = \frac{X}{\sqrt{R^2+X^2}}$$

$$\Rightarrow \tan\varphi = \frac{X}{R} \quad \Rightarrow \boxed{\varphi = \operatorname{Arctan}\frac{X}{R}}$$

* dipôle inductif : $X = L\omega > 0$

$$\Rightarrow \cos\varphi > 0 \text{ et } \sin\varphi > 0 \quad \Rightarrow \boxed{\varphi > 0}$$

1.2) $P = \langle v(t)\, i(t) \rangle = \frac{1}{2} \operatorname{Re}(\underline{v}\, \underline{i}^*)$

$$= \frac{1}{2} \operatorname{Re}\left(V_n e^{j\omega t} I_n e^{-j(\omega t - \varphi)}\right)$$

$$= \frac{1}{2} \operatorname{Re}\left(V_n I_n e^{j\varphi}\right) = \frac{V_n I_n \cos\varphi}{2}$$

$$\boxed{P = \frac{V_n I_n \cos\varphi}{2}}$$

$v(t)\, i(t) = V_M \cos\omega t\; I_M \cos(\omega t + \varphi)$
$= \dfrac{I_M V_M}{2}\left[\cos(2\omega t + \varphi) + \cos\varphi\right]$
$\langle v \cdot i \rangle = \dfrac{I_M V_M \cos\varphi}{2}$

1.3) $I_n = \dfrac{V_n}{\sqrt{R^2+X^2}} = \dfrac{320}{\sqrt{5^2+10^2}}$ $\quad \boxed{I_n = 29\,A}$

$\varphi = \operatorname{Arctan}\dfrac{X}{R} = \operatorname{Arctan}\dfrac{10}{5} = \operatorname{Arctan} 2 = 1{,}1 \text{ rad} = \boxed{63° = \varphi}$

$P = \dfrac{V_n I_n \cos\varphi}{2} \quad \Rightarrow \boxed{P = 2{,}0\,kW}$

2 - Principe du wattmètre électronique

2.1) $\langle v_2 \rangle = -\langle v_1 \rangle = -K \langle v_a v_b \rangle$

$$= -K k_a k_b \langle v(t)\, i(t) \rangle = -K k_a k_b P$$

$$\Rightarrow \boxed{k_w = -K k_a k_b}$$

2.2) $P = \dfrac{\langle v_2 \rangle}{k_w} = \dfrac{\langle v_2 \rangle}{-K k_a k_b} \quad \Rightarrow \boxed{P = 3{,}75\,kW}$

3 - Etude du moyenneur (filtre passe-bas)

Loi des nœuds en A.

3.1) en A : $V_A = \dfrac{\frac{V_1}{R} + \frac{V_2}{R} + \frac{V_B}{R} + jC_2\omega \times 0}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + jC_2\omega}$

$$\Rightarrow \boxed{V_A = \frac{V_1 + V_2 + V_B}{3 + jRC_2\omega}} \quad (1)$$

en B : (AO idéal $\Rightarrow i_+ = i_- = 0$)

$$\Rightarrow V_B = \frac{\frac{V_A}{R} + jC_1\omega V_2}{\frac{1}{R} + jC_1\omega}$$

$$\Rightarrow \boxed{V_B = \frac{V_A + jRC_1\omega V_2}{1 + jRC_1\omega}} \quad (2)$$

3.2) on a une boucle de rétroaction entre l'entrée inverseuse et la sortie de l'AO $\Rightarrow$ régime linéaire $\Rightarrow V_+ = V_-$

or $V_+ = 0$ et $V_- = V_B \Rightarrow V_B = 0$

$$(1) \Rightarrow V_A = \frac{V_1 + V_2}{3 + jRC_2\omega}$$

$$(2) \Rightarrow V_A = -jRC_1\omega V_2$$

$$\Rightarrow V_1 + V_2 = -jRC_1\omega(3 + jRC_2\omega) V_2$$

$$V_1 = \left(-1 - 3jRC_1\omega + R^2C_1C_2\omega^2\right) V_2$$

$$\boxed{\underline{T} = \frac{V_2}{V_1} = \frac{-1}{1 + 3jRC_1\omega - R^2C_1C_2\omega^2}}$$

3.3) $\underline{T} = \dfrac{T_0}{1 + \frac{1}{Q}\frac{j\omega}{\omega_0} + \left(\frac{j\omega}{\omega_0}\right)^2}$

on identifie

$\boxed{T_0 = -1}$ $\quad \omega_0^2 = \frac{1}{R^2C_1C_2} \qquad \frac{1}{Q\omega_0} = 3RC_1$

$$\Rightarrow \boxed{\omega_0 = \frac{1}{R\sqrt{C_1C_2}}} \qquad 3RC_1\omega_0 = \frac{1}{Q}$$

$$\Rightarrow \boxed{\frac{1}{Q} = 3\sqrt{\frac{C_1}{C_2}}}$$

3.4) $\frac{1}{Q\omega_0} = 3RC_1 \Rightarrow C_1 = \frac{1}{Q3R\omega_0} = \frac{1}{3R2\pi f_0 Q}$

$$\Rightarrow \boxed{C_1 = 3{,}2.10^{-8}\,F = 32\,nF}$$

$$C_2 = \frac{1}{R^2C_1\omega_0^2} = \frac{1}{4\pi^2 f_0^2 R^2 C_1} = 1{,}4.10^{-7}\,F$$

$$\boxed{C_2 = 140\,nF}$$

3.5) * $\omega \to 0$, $\underline{T} \to T_0 = -1$

$|\underline{T}| \to 1 \qquad G \to 0$

$\varphi \to \pm\pi$?

* $\omega \to \infty$, $\underline{T} \to \frac{-1}{-\frac{\omega^2}{\omega_0^2}} = \frac{\omega_0^2}{\omega^2}$

$\varphi \to 0$

$G \to 40\log\omega_0 - 40\log\omega$

pente à -20 dB/décade

* $\omega = \omega_0$, $\underline{T} = \frac{-1}{1+\frac{j}{Q}-1} = \frac{-Q}{j} = jQ$

$\varphi = \frac{\pi}{2}$ ($\Rightarrow$ pour $\omega \to 0$, $\varphi \to \pi$)

$|\underline{T}| = Q \Rightarrow G = -20\log\left(\frac{1}{Q}\right)$

$= -20\log\sqrt{2} = -3\,dB$

* intersection des asymptotes $0 = 40\log\omega_0 - 40\log\omega$

$\rightarrow$ pour $\omega = \omega_0$ ($f = f_0 = 5\,Hz$)

3.6) $\underline{T} = \frac{\underline{V_2}}{\underline{V_1}} = \frac{-1}{1+\frac{1}{\omega_0}\frac{j\omega}{Q}+\frac{1}{\omega_0^2}(j\omega)^2}$

$\underline{V_2}\left(1+\frac{1}{\omega_0 Q}j\omega + \frac{1}{\omega_0^2}(j\omega)^2\right) = -\underline{V_1}$

or $\times j\omega \Leftrightarrow \frac{d}{dt}$

$$\Rightarrow \frac{1}{\omega_0^2}\frac{d^2V_2}{dt^2} + \frac{1}{\omega_0 Q}\frac{dV_2}{dt} + V_2 = -V_1$$

$$\Rightarrow \boxed{\frac{d^2V_2}{dt^2} + \frac{\omega_0}{Q}\frac{dV_2}{dt} + \omega_0^2 V_2 = -\omega_0^2 V_1}$$

3.7) $v_1 = K v_a v_b = K k_a k_b\, v(t)\, i(t)$

$v_1 = Kk_ak_bV_nI_n\cos(\omega t)\cos(\omega t-\varphi)$

$= Kk_ak_bV_nI_n\cos\omega t\left(\cos\omega t\cos\varphi + \sin\omega t\sin\varphi\right)$

$= Kk_ak_bV_nI_n\left(\cos^2\omega t\cos\varphi + \sin\omega t\cos\omega t\sin\varphi\right)$

$= Kk_ak_bV_nI_n\left(\frac{1+\cos2\omega t}{2}\cos\varphi + \frac{1}{2}\sin2\omega t\sin\varphi\right)$

$= \frac{Kk_ak_bV_nI_n}{2}\left(\cos\varphi + \underbrace{\cos2\omega t\cos\varphi + \sin2\omega t\sin\varphi}_{\cos(2\omega t-\varphi)}\right)$

$$v_1 = \frac{Kk_ak_bV_nI_n}{2}\left(\cos\varphi + \cos(2\omega t-\varphi)\right)$$

$\Rightarrow v_1$ est donc la somme de 2 termes :

* $\frac{Kk_ak_bV_nI_n}{2}\cos\varphi$: tension continue $= \langle v_1\rangle$

* $\frac{Kk_ak_bV_nI_n}{2}\cos(2\omega t-\varphi)$: composante alternative de pulsation 2ω.

Chacune de ces tensions va donner une tension en sortie.

* tension continue : $\omega = 0 \Rightarrow \underline{T} = -1$

$\Rightarrow v_2 = -\langle v_1\rangle$

* composante alternative : pulsation $2\omega = 200\pi$ rad.s^{-1}

$\Rightarrow \frac{\omega}{\omega_0} = \frac{200\pi}{10\pi} = 20$

$\Rightarrow |\underline{T}| = \frac{1}{\sqrt{(1-20)^2 + 4m^2 \times 20^2}} = 2{,}9\ 10^{-2}$

$A_n = |\underline{T}| \times \frac{Kk_ak_bV_nI_n}{2} = \boxed{3{,}2.10^{-2}\,V = A_n}$

$\frac{A_n}{\langle v_1\rangle} = \frac{|\underline{T}|(2\omega = 200\pi)}{\cos\varphi} = \frac{2{,}9.10^{-2}}{\cos 1} = 5{,}4.10^{-2} = \boxed{5{,}4\%}$

précision relative de la mesure de P