Correction des exercices d'entrainement trigo

Correction 1 1. On sait que $\frac{5\pi}{4} \equiv -\frac{3\pi}{4}[2\pi]$ et $\frac{7\pi}{4} \equiv -\frac{\pi}{4}[2\pi]$ donc

$$\cos\frac{\pi}{4} + \cos\frac{3\pi}{4} + \cos\frac{5\pi}{4} + \cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = 0$$

2. On sait que $\frac{5\pi}{6} = \pi - \frac{\pi}{6}$ et $\frac{7\pi}{6} = \pi + \frac{\pi}{6}$ donc $\sin\frac{5\pi}{6} + \sin\frac{7\pi}{6} = \frac{1}{2} - \frac{1}{2} = 0$

3. On a

   - $\tan\frac{2\pi}{3} = -\sqrt{3}$.
   - $\tan\frac{3\pi}{4} = -1$
   - $\tan\frac{5\pi}{6} = -\frac{1}{\sqrt{3}}$ et
   - $\tan\frac{7\pi}{6} = \frac{1}{\sqrt{3}}$.

   On a donc $\tan\frac{2\pi}{3} + \tan\frac{3\pi}{4} + \tan\frac{5\pi}{6} + \tan\frac{7\pi}{6} = -\left(\sqrt{3}+1\right)$.

4. $\cos^2\frac{4\pi}{3} - \sin^2\frac{4\pi}{3} = \cos\frac{8\pi}{3} = \cos\left(2\pi + \frac{2\pi}{3}\right) = -\frac{1}{2}$. Si on y va directement, on a $\cos\frac{4\pi}{3} = -\frac{1}{2}$ et $\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$ donc $\cos^2\frac{4\pi}{3} - \sin^2\frac{4\pi}{3} = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$ et on retrouve le même résultat.

Correction 2 1. $\sin(\pi - x) + \cos\left(\frac{\pi}{2} + x\right) = \sin(x) - \sin(x) = 0$

2. $\sin(x-\pi) + \cos(\pi + x) + \sin\left(\frac{\pi}{2} - x\right) = -\sin(x) - \cos(x) + \cos(x) = -\sin(x)$.

3. $\sin\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} + x\right) = \cos(x) + \cos(x) = 2\cos(x)$.

4. $\cos(x-\pi) + \sin\left(-\frac{\pi}{2} - x\right) = -\cos(x) - \cos(x) = -2\cos(x)$

Correction 3 1. On écrit $\frac{5\pi}{12} = \frac{(2+3)\pi}{12} = \frac{\pi}{6} + \frac{\pi}{4}$, on a donc

$$\cos\frac{5\pi}{12} = \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}(\sqrt{3}-1)$$

2. On peut écrire $\frac{\pi}{12} = \frac{(4-3)\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$, ou bien $\frac{\pi}{12} = \frac{(3-2)\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$. Dans les deux cas, on obtient $\cos\frac{\pi}{12} = \frac{\sqrt{2}}{4}(\sqrt{3}+1)$.

3. On peut faire la même décomposition qu'à la question précédente ou bien on remarque que $\frac{\pi}{12} = \frac{\pi}{2} - \frac{5\pi}{12}$ donc

$$\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{2} - \frac{5\pi}{6}\right) = \cos\frac{5\pi}{6} = \frac{\sqrt{2}}{4}(\sqrt{3}-1).$$

4. On peut écrire $\frac{7\pi}{12} = \frac{(3+4)\pi}{12} = \frac{\pi}{4} + \frac{\pi}{3}$ ou bien on remarque que $\frac{7\pi}{12} = \pi - \frac{5\pi}{12}$ donc $\cos\frac{7\pi}{12} = \cos\left(\pi - \frac{5\pi}{12}\right) = -\cos\frac{5\pi}{12} = \frac{\sqrt{2}}{4}(1-\sqrt{3})$.

5. On écrit $\cos\frac{\pi}{4} = \cos\left(2\frac{\pi}{8}\right) = 2\cos^2\frac{\pi}{8} - 1$. On en déduit que $\cos^2\frac{\pi}{8} = \frac{\sqrt{2}+2}{4}$, puis comme $\cos\frac{\pi}{8} > 0$, $\cos\frac{\pi}{8} = \frac{\sqrt{\sqrt{2}+2}}{2}$.

6. On écrit $\cos\frac{\pi}{4} = \cos\left(2\frac{\pi}{8}\right) = 1 - 2\sin^2\frac{\pi}{8}$, on en déduit $\sin^2\frac{\pi}{8} = \frac{2-\sqrt{2}}{4}$, puis, comme $\sin\frac{\pi}{8} > 0$, on a $\sin\frac{\pi}{8} = \frac{\sqrt{2-\sqrt{2}}}{2}$.

Correction 4 1. $\cos(3x)\sin(x) - \sin(3x)\cos(x) = \sin(x-3x) = -\sin(2x)$.

2. $\cos(x)\cos(4x) + \sin(x)\sin(4x) = \cos(4x - x) = \cos(3x)$.

3. On a

   - $\cos\left(x + \frac{2\pi}{3}\right) = \cos(x)\cos\frac{2\pi}{3} - \sin(x)\sin\frac{2\pi}{3}$ et
   - $\cos\left(x + \frac{4\pi}{3}\right) = \cos(x)\cos\frac{4\pi}{3} - \sin(x)\sin\frac{4\pi}{3}$.

Comme $\sin\frac{2\pi}{3} = -\sin\frac{4\pi}{3}$ et $\cos\frac{2\pi}{3} = \cos\frac{4\pi}{3} = -\frac{1}{2}$, on a

$$\cos\left(x+\frac{2\pi}{3}\right) + \cos\left(x+\frac{4\pi}{3}\right) = -\cos(x)$$

donc

$$\cos(x) + \cos\left(x+\frac{2\pi}{3}\right) + \cos\left(x+\frac{4\pi}{3}\right) = 0$$

4. On a
   - $\cos\left(2x+\frac{\pi}{6}\right) = \cos(2x)\cos\frac{\pi}{6} - \sin(2x)\sin\frac{\pi}{6}$ et
   - $\cos\left(2x-\frac{\pi}{6}\right) = \cos(2x)\cos\frac{\pi}{6} + \sin(2x)\sin\frac{\pi}{6}$.

   On a donc

$$\begin{aligned}\cos(2x) + \cos\left(2x+\frac{\pi}{6}\right) + \cos\left(2x-\frac{\pi}{6}\right) &= \cos(2x) + 2\cos(2x)\cos\frac{\pi}{6} \\ &= \left(\sqrt{3}+1\right)\cos(2x)\end{aligned}$$

Correction 5

1. On écrit

$$\begin{aligned}\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} &= 2\cos(x) - \frac{\cos(2x)}{\cos(x)} \\ &= \frac{2\cos^2(x) - \cos(2x)}{\cos(x)} \\ &= \frac{1}{\cos(x)}\end{aligned} .$$

2. On a $\frac{1-\cos(2x)}{\sin(2x)} = \frac{2\cos^2(x)}{\sin(2x)} = \frac{2\cos^2(x)}{2\cos(x)\sin(x)} = \text{cotan}(x)$

3. On écrit

$$\begin{aligned}\frac{\sin(3x)}{\sin(x)} - \frac{\cos(3x)}{\cos(x)} &= \frac{\sin(3x)\cos(x) - \cos(3x)\sin(x)}{\sin(x)\cos(x)} \\ &= \frac{\sin(3x-x)}{\sin(x)\cos(x)} \\ &= \frac{\sin(2x)}{\cos(x)\sin(x)} \\ &= 2\end{aligned}$$