Centrifugeuse humaine (corrigé)

1.a) $\boxed{\vec{\Omega}(1/0) = \dot{\psi}\ \vec{z}_0}$ $\boxed{\vec{\Omega}(2/1) = \dot{\theta}\ \vec{x}_1}$ $\boxed{\vec{\Omega}(3/2) = \dot{\varphi}\ \vec{y}_2}$

1.b) $\vec{V}_{I\in 3/0} = \left[\dfrac{d\overrightarrow{OI}}{dt}\right]_0 = \left[\dfrac{d(-R\vec{y}_1)}{dt}\right]_0$ $\boxed{\vec{V}_{I\in 3/0} = R\ \dot{\psi}\ \vec{x}_1}$

1.c) $\boxed{\vec{\Gamma}_{I\in 3/0} = R\ \ddot{\psi}\ \vec{x}_1 + R\ \dot{\psi}^2\ \vec{y}_1}$

2.a) Solide cylindrique – Symétrie de révolution – Pas de coefficients non diagonaux - A=C

$B = \dfrac{mr^2}{2}$ et $A = \dfrac{mr^2}{4} + \dfrac{mh^2}{12}$

2.b) $\overrightarrow{R_C}_{(3/R0)} = m\vec{V}_{I\in 3/0} = m\ R\ \dot{\psi}\ \vec{x}_1$

$\vec{\sigma}_{I\,(3/R0)} = \overline{\overline{I}}(3)\ \vec{\Omega}(3/R_0)$ car I est centre d'inertie de **3** avec $\vec{\Omega}(3/R_0) = \dot{\psi}\ \vec{z}_1 + \dot{\theta}\ \vec{x}_2 + \dot{\varphi}\ \vec{y}_2$

$= \dot{\psi}\,(\cos\theta\,\vec{z}_2 + \sin\theta\,\vec{y}_2) + \dot{\theta}\ \vec{x}_2 + \dot{\varphi}\ \vec{y}_2$

$\vec{\sigma}_{I\,(3/R0)} = \begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & A \end{pmatrix} \begin{pmatrix} \dot{\theta} \\ \dot{\varphi} + \dot{\psi}\sin\theta \\ \dot{\psi}\cos\theta \end{pmatrix}$ d'où $\boxed{[Ci_{(3/R0)}] = {}_I\begin{Bmatrix} mR\,\dot{\psi}\,\vec{x}_2 \\ A\dot{\theta}\vec{x}_2 + B(\dot{\varphi} + \dot{\psi}\sin\theta)\vec{y}_2 + A\dot{\psi}\cos\theta\vec{z}_2 \end{Bmatrix}}$

2.c) $\overrightarrow{R_D}_{(3/R0)} = m\ \vec{\Gamma}_{I\in 3/0} = m\,(R\ \ddot{\psi}\ \vec{x}_1 + R\ \dot{\psi}^2\ \vec{y}_1)$

$\vec{\delta}_{I\,(3/R0)} = \left[\dfrac{d\vec{\sigma}_{I(3/R_0)}}{dt}\right]_{R_0}$ car I est centre d'inertie de **3**

$= \left[\dfrac{d\vec{\sigma}_{I(3/R_0)}}{dt}\right]_{R_2} + \vec{\Omega}(R_2/R_0) \wedge \vec{\sigma}_{I\,(3/R0)}$ avec $\vec{\Omega}(R_2/R_0) = \dot{\psi}\ \vec{z}_1 + \dot{\theta}\ \vec{x}_2 = \dot{\psi}\,(\cos\theta\,\vec{z}_2 + \sin\theta\,\vec{y}_2) + \dot{\theta}\ \vec{x}_2$

$$\vec{\delta}_{I(3/R0)} = A\ddot{\theta}\vec{x}_2 + B(\ddot{\varphi} + \ddot{\psi}\sin\theta + \dot{\psi}\dot{\theta}\cos\theta)\vec{y}_2 + \left(A\ddot{\psi}\cos\theta - A\dot{\psi}\dot{\theta}\sin\theta\right)\vec{z}_2$$

$$+A\dot{\theta}\left(\frac{d\vec{x}_2}{dt}\right)_0 + B(\dot{\varphi} + \dot{\psi}\sin\theta)\left(\frac{d\vec{y}_2}{dt}\right)_0 + A\dot{\psi}\cos\theta\left(\frac{d\vec{z}_2}{dt}\right)_0$$

$$\vec{\delta}_{I(3/R0)} = A\ddot{\theta}\vec{x}_2 + B(\ddot{\varphi} + \ddot{\psi}\sin\theta + \dot{\psi}\dot{\theta}\cos\theta)\vec{y}_2 + \left(A\ddot{\psi}\cos\theta - A\dot{\psi}\dot{\theta}\sin\theta\right)\vec{z}_2$$

$$+A\dot{\theta}\left(\frac{d\vec{x}_2}{dt}\right)_0 + B(\dot{\varphi} + \dot{\psi}\sin\theta)\left(\frac{d\vec{y}_2}{dt}\right)_0 + A\dot{\psi}\cos\theta\left(\frac{d\vec{z}_2}{dt}\right)_0$$

$$\left(\frac{d\vec{x}_2}{dt}\right)_0 = \left(\frac{d\vec{x}_1}{dt}\right)_0 = \left(\frac{d\vec{x}_1}{dt}\right)_1 + \left(\dot{\psi}\vec{z}_1\right) \wedge \vec{x}_1 = \dot{\psi}\vec{y}_1$$

$$\left(\frac{d\vec{y}_2}{dt}\right)_0 = \left(\frac{d\vec{y}_2}{dt}\right)_2 + \left(\dot{\psi}\vec{z}_1 + \dot{\theta}\vec{x}_2\right) \wedge \vec{y}_2 = \dot{\theta}\vec{z}_2 - \dot{\psi}\sin\left(\frac{\pi}{2} - \theta\right)\vec{x}_2 = \dot{\theta}\vec{z}_2 - \dot{\psi}\cos\theta\vec{x}_2$$

$$\left(\frac{d\vec{z}_2}{dt}\right)_0 = \left(\frac{d\vec{z}_2}{dt}\right)_2 + \left(\dot{\psi}\vec{z}_1 + \dot{\theta}\vec{x}_2\right) \wedge \vec{z}_2 = \dot{\psi}\sin\theta\vec{x}_2 - \dot{\theta}\vec{y}_2$$

$$\vec{\delta}_{I(3/R0)} = A\ddot{\theta}\vec{x}_2 + B(\ddot{\varphi} + \ddot{\psi}\sin\theta + \dot{\psi}\dot{\theta}\cos\theta)\vec{y}_2 + \left(A\ddot{\psi}\cos\theta - A\dot{\psi}\dot{\theta}\sin\theta\right)\vec{z}_2$$
$$+A\dot{\theta}\dot{\psi}\vec{y}_1 + B(\dot{\varphi} + \dot{\psi}\sin\theta)\left(\dot{\theta}\vec{z}_2 - \dot{\psi}\cos\theta\vec{x}_2\right) + A\dot{\psi}\cos\theta\left(\dot{\psi}\sin\theta\vec{x}_2 - \dot{\theta}\vec{y}_2\right)$$

$$\vec{y}_1 = \cos\theta\vec{y}_2 - \sin\theta\vec{z}_2$$

d'où
$$\boxed{[D_{(3/R0)}] = {}_I\left\{\begin{array}{c|c} m R \ddot{\psi} & A\ddot{\theta} + A\dot{\psi}^2\sin\theta\cos\theta - B(\dot{\varphi} + \dot{\psi}\sin\theta)\dot{\psi}\cos\theta \\ m R \dot{\psi}^2\cos\theta & B(\ddot{\varphi} + \ddot{\psi}\sin\theta + \dot{\psi}\dot{\theta}\cos\theta) \\ -m R \dot{\psi}^2\sin\theta & A\ddot{\psi}\cos\theta + (B-2A)\dot{\psi}\dot{\theta}\sin\theta + B\dot{\varphi}\dot{\theta} \end{array}\right\}_{B_2}}$$

2.d) Bilan des actions sur **3** :

$\mathsf{T}(pes\to3) = {}_I\left\{\begin{array}{c} m g \vec{z}_0 \\ \vec{0} \end{array}\right\}$ $\quad \mathsf{T}(mot\to3) = {}_I\left\{\begin{array}{c} \vec{0} \\ C_{23}\vec{y}_2 \end{array}\right\}$ $\quad \mathsf{T}(2\to3)$ donné

2.e) Isolement de 3 -TMD en I selon $\vec{y}_2$

2.f-g)

Principe fondamental de la dynamique appliqué à **3** : $[D_{(3/R0)}] = \mathsf{T}(pes\to3) + \mathsf{T}(mot\to3) + \mathsf{T}(2\to3)$

Théorème de la résultante dynamique :

sur $\vec{x}_2$: $m R \ddot{\psi} = X_{23}$

sur $\vec{y}_2$: $m R \dot{\psi}^2\cos\theta = m g \sin\theta + Y_{23}$

sur $\vec{z}_2$: $-m R \dot{\psi}^2\sin\theta = m g \cos\theta + Z_{23}$

Théorème de la moment dynamique :

sur $\vec{x}_2$: $A\ddot{\theta} + A\dot{\psi}^2\sin\theta\cos\theta - B(\dot{\varphi} + \dot{\psi}\sin\theta)\dot{\psi}\cos\theta = L_{23}$

sur $\vec{y}_2$: $\boxed{B(\ddot{\varphi} + \ddot{\psi}\sin\theta + \dot{\psi}\dot{\theta}\cos\theta) = C_{23}}$

sur $\vec{z}_2$: $A\ddot{\psi}\cos\theta + (B-2A)\dot{\psi}\dot{\theta}\sin\theta + B\dot{\varphi}\dot{\theta} = N_{23}$