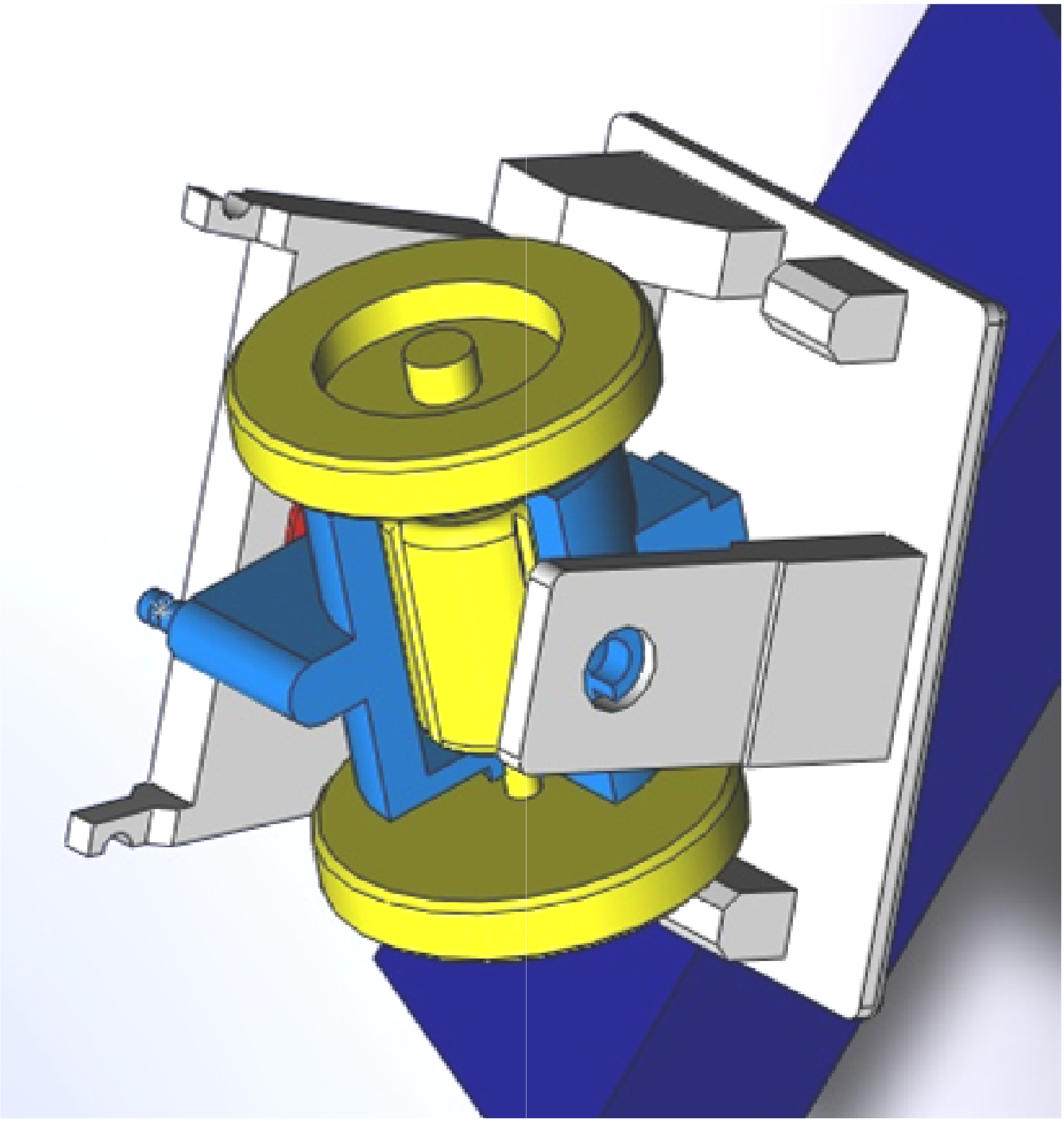
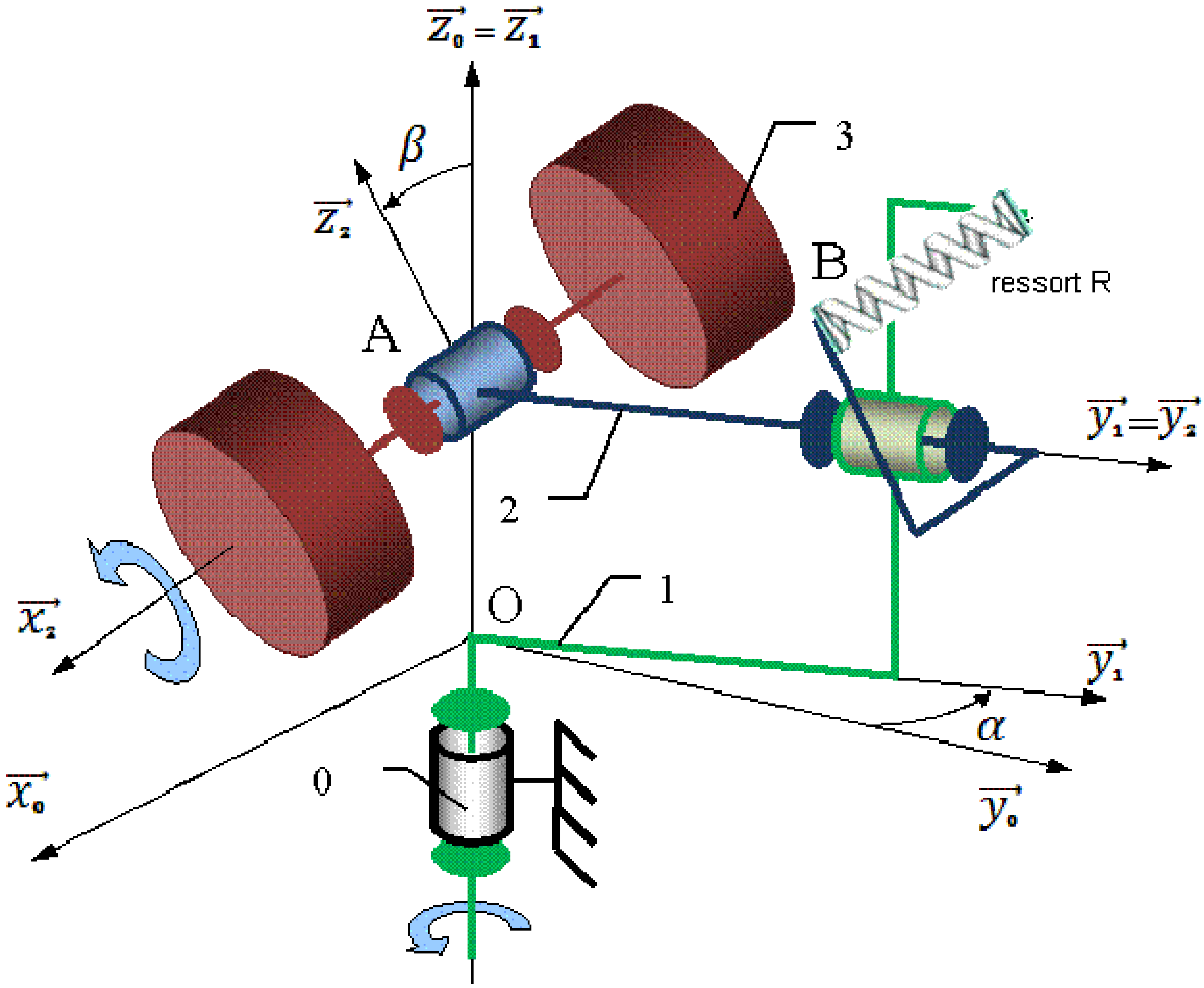
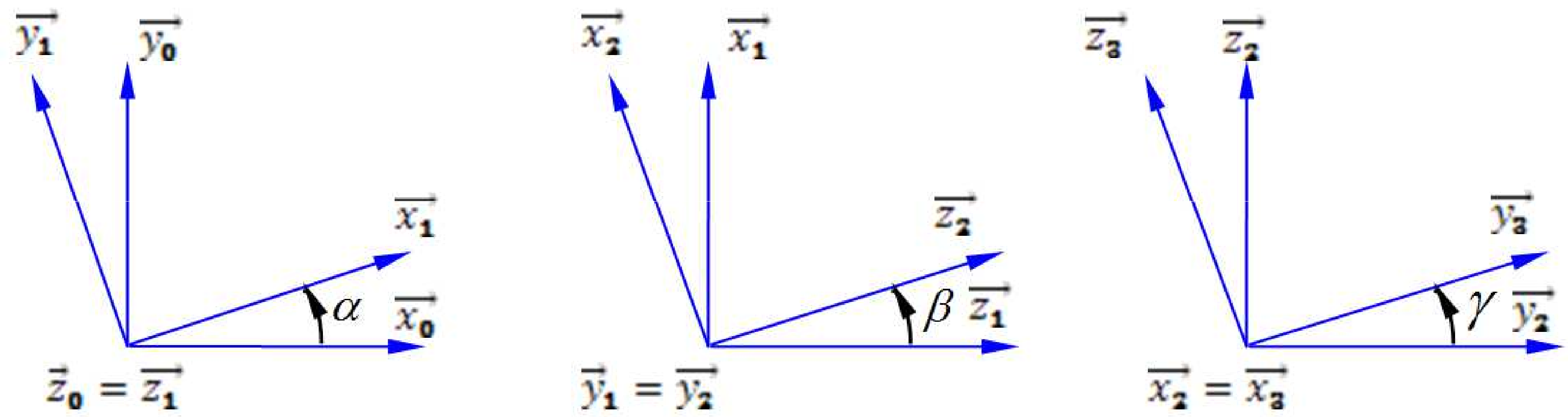

Rotor Principal P
Rotor Anti-Couple AC
Queue
Axe de LACET

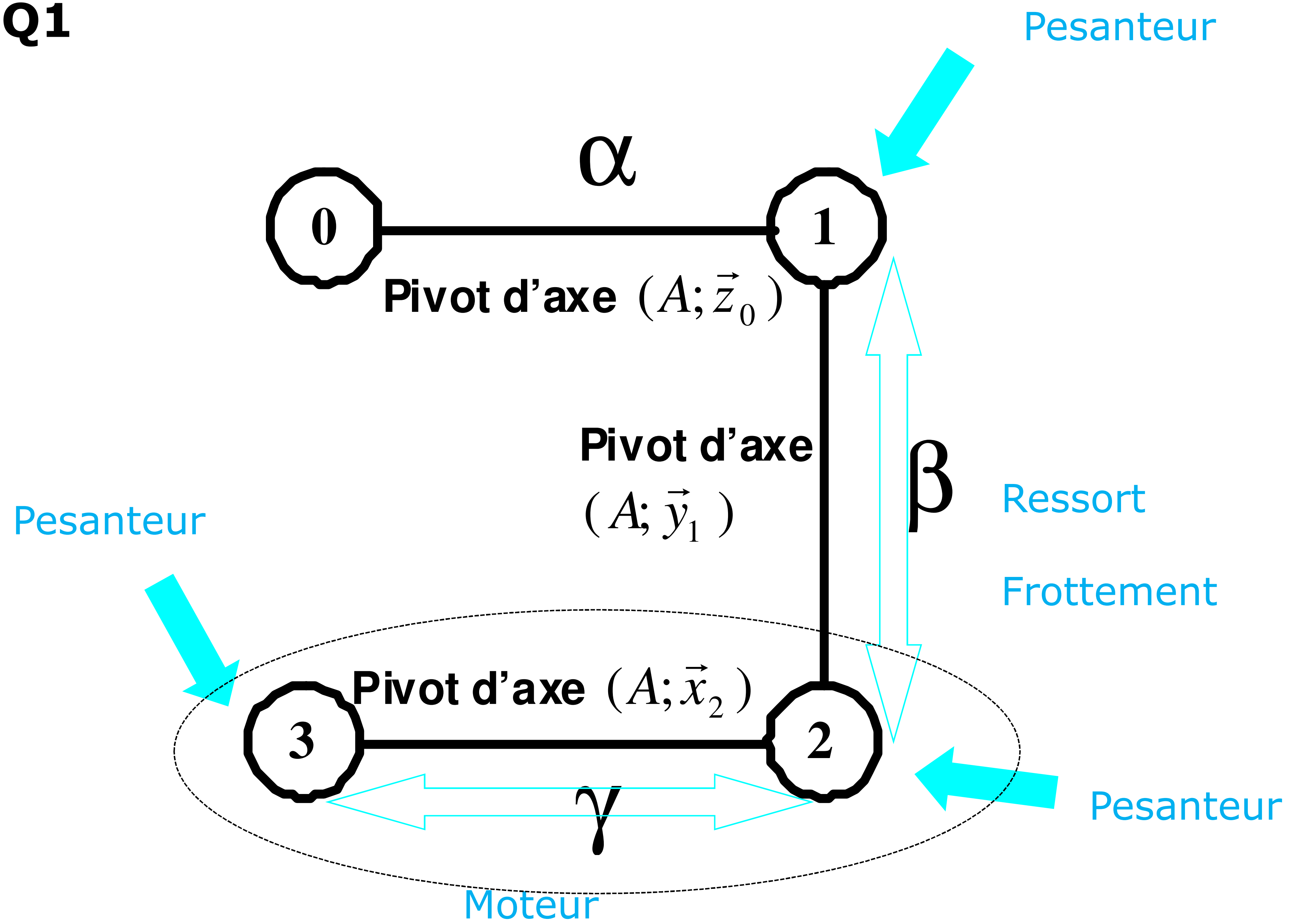



$\vec{z_0} = \vec{z_1}$
β
$\vec{z_2}$
3
A
B
ressort R
$\vec{y_1} = \vec{y_2}$
2
$\vec{x_2}$
O
1
$\vec{y_1}$
α
0
$\vec{x_0}$
$\vec{y_0}$

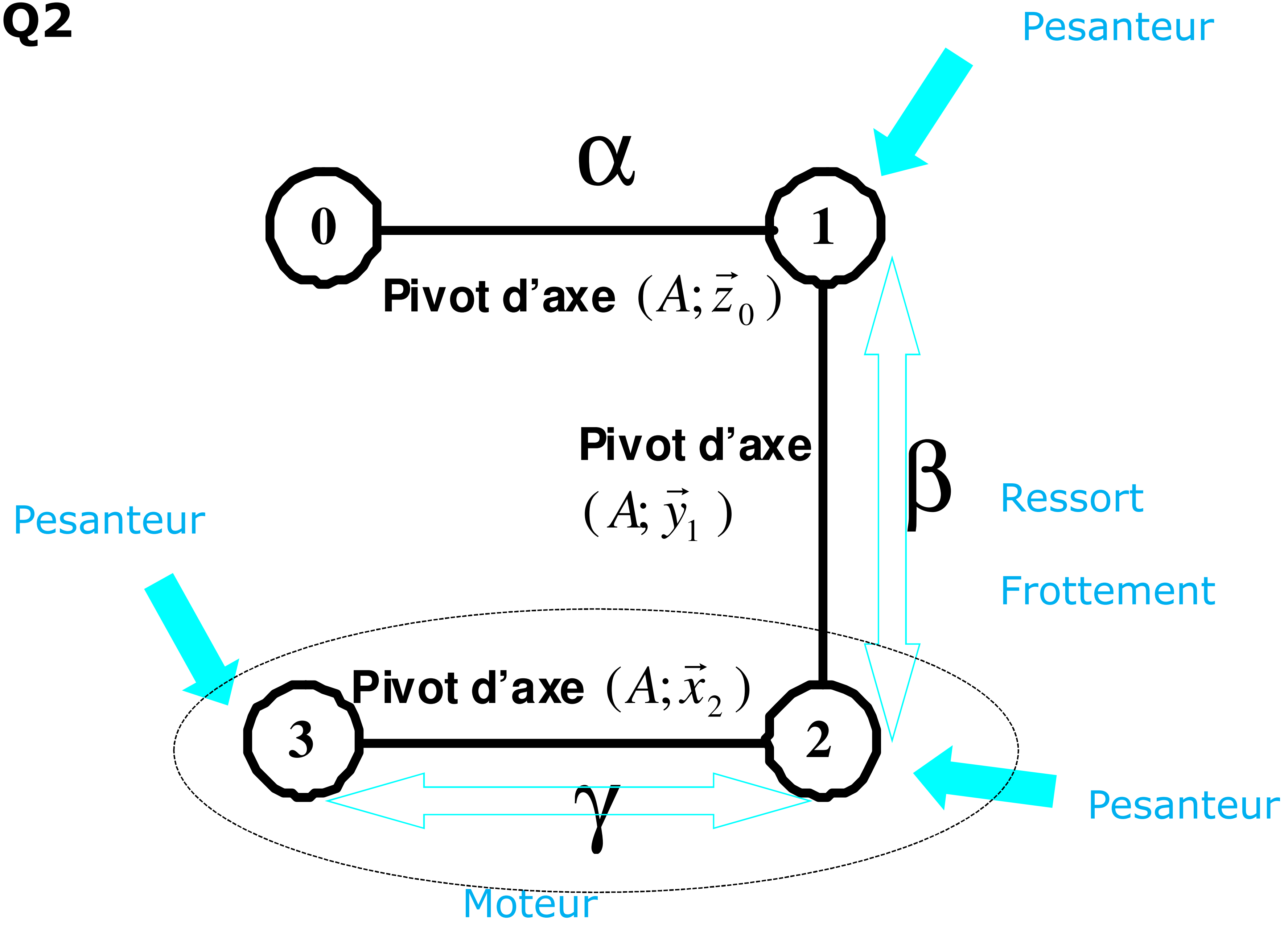


$\overrightarrow{y_1}$
$\overrightarrow{y_0}$
$\overrightarrow{x_1}$
α
$\overrightarrow{x_0}$
$\vec{z}_0 = \overrightarrow{z_1}$
$\overrightarrow{x_2}$
$\overrightarrow{x_1}$
$\overrightarrow{z_2}$
β
$\overrightarrow{z_1}$
$\vec{y}_1 = \overrightarrow{y_2}$
$\overrightarrow{z_3}$
$\overrightarrow{z_2}$
$\overrightarrow{y_3}$
γ
$\overrightarrow{y_2}$
$\overrightarrow{x_2} = \overrightarrow{x_3}$

Q1


Pesanteur
α
0
1
Pivot d'axe $(A;\vec{z}_0)$
Pivot d'axe
$(A;\vec{y}_1)$
β
Ressort
Frottement
Pesanteur
Pivot d'axe $(A;\vec{x}_2)$
3
2
γ
Pesanteur
Moteur

Q2


Pesanteur
α
0
1
Pivot d'axe $(A;\vec{z}_0)$
Pivot d'axe
$(A;\vec{y}_1)$
β
Ressort
Frottement
Pesanteur
Pivot d'axe $(A;\vec{x}_2)$
3
2
γ
Pesanteur
Moteur

Q2

Principe fondamental de la dynamique appliqué aux solides (2) et (3) écrite **en A** et en projection sur $\vec{y}_2$

$$\vec{\delta}(A, 2+3/0).\vec{y}_2 = \vec{M}(A, pes \rightarrow 2+3).\vec{y}_2$$

$$+\vec{M}(A, \underbrace{1 \rightarrow 2}_{\text{pivot+frottement}}).\vec{y}_2 + \vec{M}(A, \underbrace{1 \rightarrow 2}_{\text{Re ssort}}).\vec{y}_2$$

Q3

BAME

$$\left\{T_{\underbrace{1\to2}_{ressort}}\right\} = {}_{B}\begin{Bmatrix} -hk\sin\beta & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_{(\vec{x}_2,\vec{y}_2,\vec{z}_2)}$$

$$\left\{T_{\underbrace{1\to2}_{pivot}}\right\} = {}_{A}\begin{Bmatrix} X_{12} & L_{12} \\ Y_{12} & 0 \\ Z_{12} & N_{12} \end{Bmatrix}_{(\vec{x}_2,\vec{y}_2,\vec{z}_2)} \to \left\{T_{\underbrace{1\to2}_{pivot+frottement}}\right\} = {}_{A}\begin{Bmatrix} X_{12} & L_{12} \\ Y_{12} & -f\dot{\beta} \\ Z_{12} & N_{12} \end{Bmatrix}_{(\vec{x}_2,\vec{y}_2,\vec{z}_2)}$$

$$\left\{T_{pes\to2+3}\right\} = {}_{A}\begin{Bmatrix} 0 & 0 \\ 0 & 0 \\ -(m_3+m_2)g & 0 \end{Bmatrix}_{(\vec{x}_1,\vec{y}_1,\vec{z}_1)}$$

Q3

BAME

$$\left\{ T_{\underbrace{1\to 2}_{ressort}} \right\} = {}_B\left\{ \begin{matrix} -hk\sin\beta & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix} \right\}_{\left(\vec{x}_2,\vec{y}_2,\vec{z}_2\right)}$$

$$\vec{M}(A,\underbrace{1 \to 2}_{\text{ressort}}).\vec{y}_2 = \left(-hk\sin\beta.\vec{x}_2 \wedge \overrightarrow{BA}\right).\vec{y}_2$$

$$\overrightarrow{AB}.\vec{z}_2 = h$$

Q3

BAME

$$\left\{T_{\underbrace{1\to 2}_{ressort}}\right\} = {}_{B}\begin{Bmatrix} -hk\sin\beta & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_{(\vec{x}_2,\vec{y}_2,\vec{z}_2)}$$

$$\vec{M}(A,\underbrace{1\to 2}_{\text{ressort}}).\vec{y}_2 = \left(-hk\sin\beta.\vec{x}_2 \wedge \overrightarrow{BA}\right).\vec{y}_2$$

$$= \left(\vec{y}_2 \wedge -hk\sin\beta.\vec{x}_2\right).\overrightarrow{BA}$$

$$= \left(hk\sin\beta.\vec{z}_2\right).\overrightarrow{BA} = -h^2k\sin\beta$$

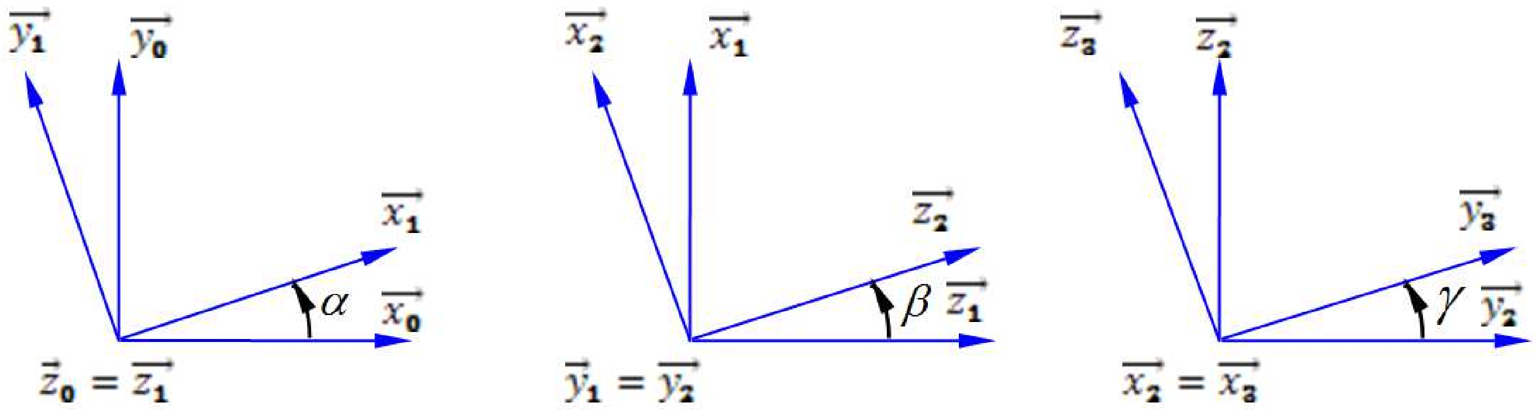
$$\overrightarrow{AB}.\vec{z}_2 = h$$

Q3

BAME

$$\left\{T_{pes\rightarrow 2+3}\right\} = {}_{A}\begin{Bmatrix} 0 & 0 \\ 0 & 0 \\ -(m_3+m_2)\,g & 0 \end{Bmatrix}_{(\vec{x}_1,\vec{y}_1,\vec{z}_1)}$$

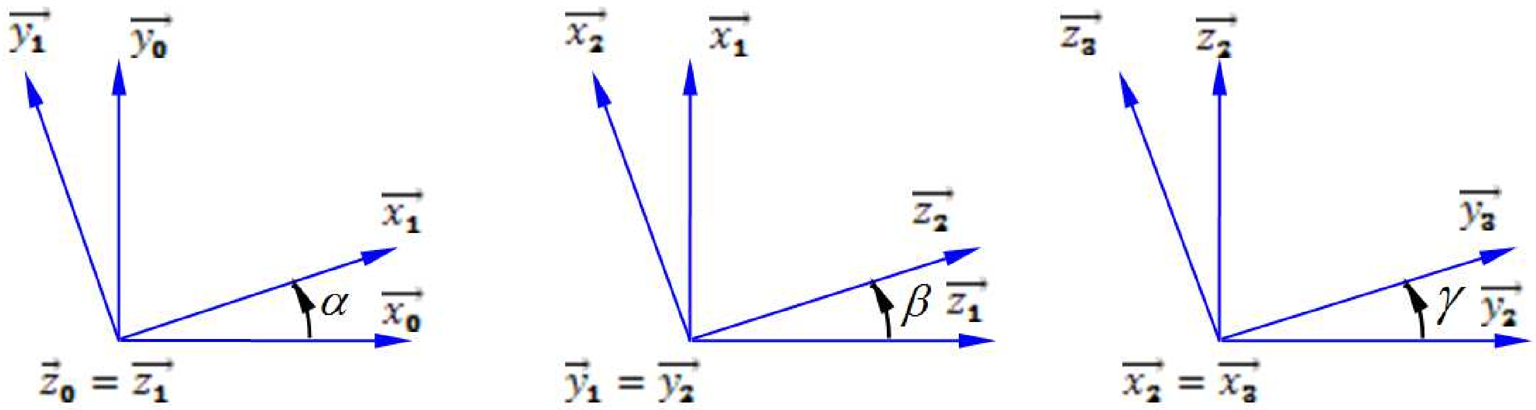
$$\vec{M}(A,\text{pes} \rightarrow 2+3).\vec{y}_2 = \vec{0}.\vec{y}_2 = 0$$



Q3 Calcul de $\vec{\delta}(A, 2+3/0).\vec{y}_2$

$$\vec{\sigma}(A, 2+3/0) = \vec{\sigma}(A, 3/0) = I(A, 3).\vec{\Omega}_{3/0}$$

$$\vec{\Omega}_{3/0} = \dot{\alpha}.\vec{z}_1 + \dot{\beta}.\vec{y}_2 + \dot{\gamma}.\vec{x}_2$$



Q3 Calcul de $\vec{\delta}(A,2+3/0).\vec{y}_2$

$$\vec{\sigma}(A,2+3/0)=\vec{\sigma}(A,3/0)=I(A,3).\vec{\Omega}_{3/0}$$

$$\vec{\Omega}_{3/0}=\dot{\alpha}.\vec{z}_1+\dot{\beta}.\vec{y}_2+\dot{\gamma}.\vec{x}_2$$

$$\vec{\Omega}_{3/0}=(\dot{\gamma}-\dot{\alpha}\sin\beta).\vec{x}_2+\dot{\beta}.\vec{y}_2+\dot{\alpha}\cos\beta.\vec{z}_2$$

Q3 Calcul de $\vec{\delta}(A,2+3/0).\vec{y}_2$

$$\vec{\sigma}(A,2+3/0)=\vec{\sigma}(A,3/0)=I(A,3).\vec{\Omega}_{3/0}$$

$$\vec{\Omega}_{3/0}=\dot{\alpha}.\vec{z}_1+\dot{\beta}.\vec{y}_2+\dot{\gamma}.\vec{x}_2$$

$$\vec{\Omega}_{3/0}=(\dot{\gamma}-\dot{\alpha}\sin\beta).\vec{x}_2+\dot{\beta}.\vec{y}_2+\dot{\alpha}\cos\beta.\vec{z}_2$$

$$\vec{\sigma}(A,3/0)=A(\dot{\gamma}-\dot{\alpha}\sin\beta).\vec{x}_2+B\dot{\beta}.\vec{y}_2+B\dot{\alpha}\cos\beta.\vec{z}_2$$

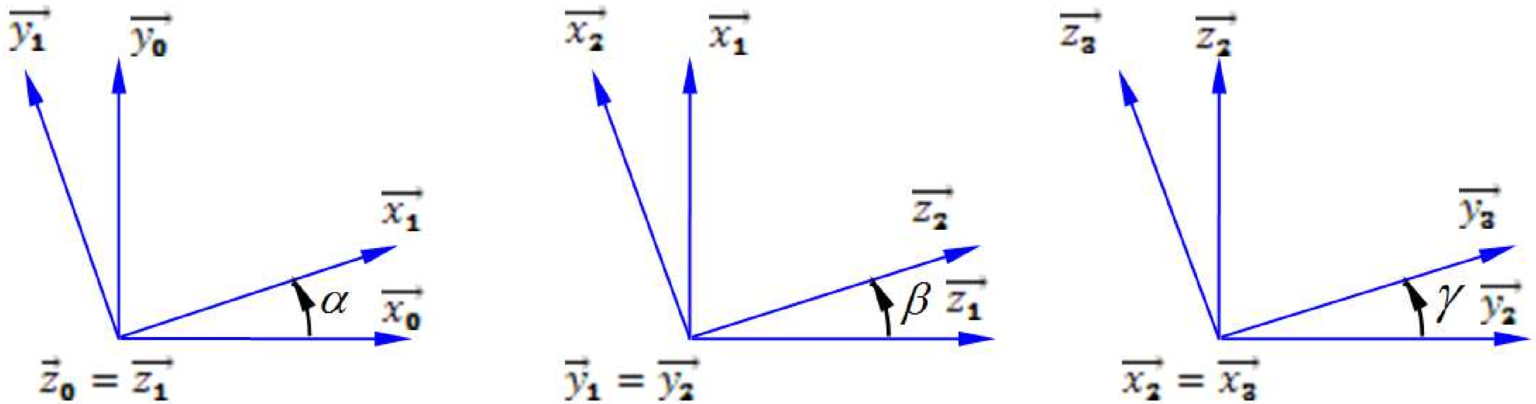
Q3 Calcul de $\vec{\delta}(A, 2+3/0).\vec{y}_2$

$$\vec{\delta}(A,3/0) = \left(\frac{d\vec{\sigma}(A,3/0)}{dt}\right)_{R0}$$

$$\vec{\delta}(A,3/0).\vec{y}_2 = \left(\frac{d[\vec{\sigma}(A,3/0).\vec{y}_2]}{dt}\right)_{R0} - \vec{\sigma}(A,3/0)\left(\frac{d\vec{y}_2}{dt}\right)_{R0}$$

Q3 Calcul de $\vec{\delta}(A, 2+3/0).\vec{y}_2$

$$\left(\frac{d\vec{y}_2}{dt}\right)_{R0} = \left(\frac{d\vec{y}_1}{dt}\right)_{R0} = \left(\frac{d\vec{y}_1}{dt}\right)_{R1} + \dot{\alpha}.\vec{z}_1 \wedge \vec{y}_1 = -\dot{\alpha}\vec{x}_1$$

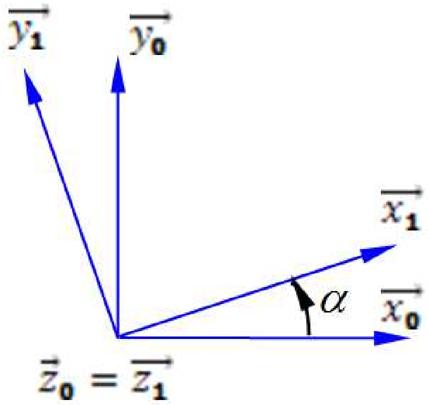


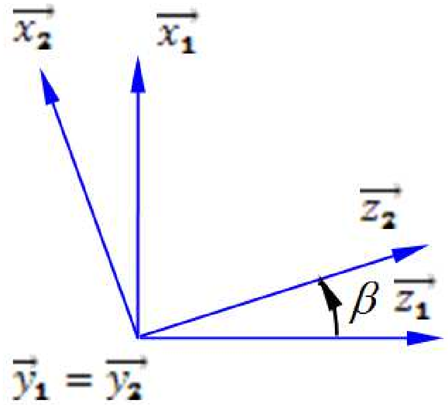
Q3 Calcul de $\vec{\delta}(A, 2+3/0).\vec{y}_2$

$$\vec{\delta}(A,3/0).\vec{y}_2 = \left(\frac{d[\vec{\sigma}(A,3/0).\vec{y}_2]}{dt}\right)_{R0} - \vec{\sigma}(A,3/0)\left(\frac{d\vec{y}_2}{dt}\right)_{R0}$$

$$\vec{\sigma}(A,3/0) = A\ \ (\dot{\gamma} - \dot{\alpha}\sin\beta).\vec{x}_2 + B\dot{\beta}.\vec{y}_2 + B\dot{\alpha}\cos\beta.\vec{z}_2$$

$$\left(\frac{d\vec{y}_2}{dt}\right)_{R0} = -\dot{\alpha}\vec{x}_1$$





Q3 Calcul de $\vec{\delta}(A, 2+3/0).\vec{y}_2$

$$\vec{\delta}(A,3/0).\vec{y}_2 = \left(\frac{d[\vec{\sigma}(A,3/0).\vec{y}_2]}{dt}\right)_{R0} - \vec{\sigma}(A,3/0)\left(\frac{d\vec{y}_2}{dt}\right)_{R0}$$

$$= B\ddot{\beta} + A\dot{\alpha}\dot{\gamma}\cos\beta + (B-A)\dot{\alpha}^2\sin\beta\cos\beta$$

Rappel

$$\vec{M}(A,\underbrace{1 \rightarrow 2}_{\text{ressort}}).\vec{y}_2 = \left(hk\sin\beta.\vec{z}_2\right).\overrightarrow{BA} = -h^2k\sin\beta$$

$$\vec{M}(A,\underbrace{1 \rightarrow 2}_{\text{pivot+frottement}}).\vec{y}_2 = -f\dot{\beta}$$

Q3 Calcul de $\vec{\delta}(A, 2+3/0).\vec{y}_2$

$$\boxed{B\ddot{\beta} + A\dot{\alpha}\dot{\gamma}\cos\beta + (B-A)\dot{\alpha}^2\sin\beta\cos\beta = -k\sin\beta\, h^2 - f\,\dot{\beta}}$$

Q4 Conditions sur l'angle $-9° < \beta < 9°$

$$B\ddot{\beta} + A\dot{\alpha}\dot{\gamma}\cos\beta + (B-A)\dot{\alpha}^2 \sin\beta\cos\beta = -k\sin\beta\, h^2 - f\,\dot{\beta}$$

$$\Downarrow$$

$$\boxed{B\ddot{\beta} + A\dot{\alpha}\dot{\gamma} + (B-A)\dot{\alpha}^2 \beta = -k\,\beta\, h^2 - f\,\dot{\beta}}$$

Q4 $\dot{\gamma} >> \dot{\alpha}$ $B \approx A$ $-9^\circ < \beta < 9^\circ$

$$B\ddot{\beta} + A\dot{\alpha}\dot{\gamma} + (B - A)\dot{\alpha}^2\beta = -k\beta h^2 - f\dot{\beta}$$

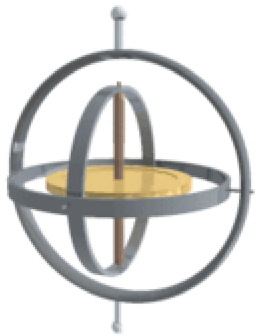
$$A\dot{\alpha}\dot{\gamma} \gg (B - A)\dot{\alpha}^2\beta$$

$$\boxed{B\ddot{\beta} + A\dot{\alpha}\dot{\gamma} = -k\beta h^2 - f\dot{\beta}}$$

Q4 Remarques : si $A\ faible\ ou\ \dot{\alpha}=0\ ou\ \dot{\gamma}=0$

$$B\ddot{\beta}+f\dot{\beta}+k\beta h^2=0$$

$A\dot{\alpha}\dot{\gamma}$ **: couple gyroscopique → moto , vélo , toupie**



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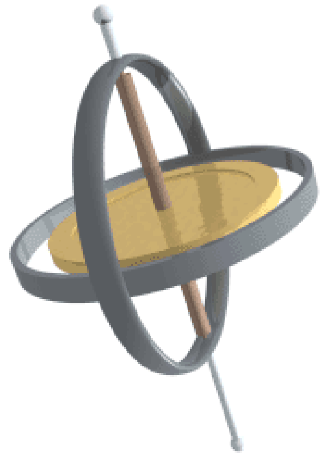

Q5 Transformée de Laplace

$$(Bp^2 + fp + kh^2)\beta(p) = -A\dot{\gamma}\Omega(p)$$

$$H(p) = \frac{A\dot{\gamma}/kh^2}{1 + \frac{f}{kh^2}p + \frac{B}{kh^2}p^2}$$

Q5 Transformée de Laplace

$$-\Omega(p) \longrightarrow \boxed{\dfrac{A\dot{\gamma}/kh^2}{1+\dfrac{f}{kh^2}\,p+\dfrac{B}{kh^2}\,p^2}} \longrightarrow \beta(p)$$

$$\omega_0 = h\sqrt{\frac{k}{B}} \qquad\qquad \xi = \frac{f}{2h\sqrt{kB}}$$

Q6 Transformée de Laplace

$$-\Omega(p) \longrightarrow \boxed{\dfrac{{}^{A\dot{\gamma}}\!/\!_{kh^2}}{1+\dfrac{f}{kh^2}\,p+\dfrac{B}{kh^2}\,p^2}} \longrightarrow \beta(p)$$

$$\omega_0 = h\sqrt{\frac{k}{B}} = 86 rad.s^{-1}$$

$$\omega_0 = h\sqrt{\frac{k}{B}} = 86 rad.s^{-1} \rightarrow k = B\left(\frac{\omega_0}{h}\right)^2 = 350 N/m$$

Q7 Transformée de Laplace

$$-\Omega(p) \longrightarrow \boxed{\dfrac{A\dot{\gamma}/kh^2}{1+\dfrac{f}{kh^2}p+\dfrac{B}{kh^2}p^2}} \longrightarrow \beta(p)$$

$$\xi = \frac{f}{2h\sqrt{kB}} = 0,7$$

$$\xi = \frac{f}{2h\sqrt{kB}} = 0,7 \rightarrow f = 2\xi h\sqrt{kB} = 7.5.10^{-4}\,N.m.s$$

Q8 Transformée de Laplace

$$-\Omega(p) \longrightarrow \boxed{\dfrac{A\dot{\gamma}/kh^2}{1+\dfrac{f}{kh^2}p+\dfrac{B}{kh^2}p^2}} \longrightarrow \beta(p)$$

$$\omega(t) = \dot{\alpha}_0 u(t) \qquad \lim_{t\to\infty} \beta(t) = \frac{-A\dot{\gamma}}{kh^2}\dot{\alpha}_0$$

$$\beta(\infty) = \frac{-A\dot{\gamma}}{kh^2}\dot{\alpha}_0$$