CORRIGÉ DS n°4

I ZrC

1) $\Delta_r H^\circ = \Delta_f H^\circ(CO_2) + \Delta_f H^\circ(ZrO_2) - \Delta_f H^\circ(ZrC)$

$= -400 - 1100 + 200 = -1300 \text{ kJ/mol} < 0$

exothermique.

2) $\Delta_r S^\circ = 210 + 50 - 2\times 200 - 30 = -170 \text{ J.K}^{-1}.\text{mol}^{-1} < 0$

logique car globalement la réaction détruit du gaz, donc le désordre diminue.

3) $\Delta_r G^\circ = \Delta_r H^\circ - T\Delta_r S^\circ = -1300\,10^3 + 300\times 170 = -1{,}3\,10^6 + 5{,}1\,10^5$

$= -0{,}8\,10^6 \text{ kJ/mol}$

$= -8\,10^5 \text{ kJ/mol}$

4) $K^\circ = \exp -\dfrac{\Delta_r G^\circ}{RT}$

$= \exp + \dfrac{8\,10^5}{8{,}3\times 300} = \exp \dfrac{10^3}{3} >> 1$

→ réaction totale.

5) Van't Hoff $\dfrac{d \ln K^\circ}{dT} = \dfrac{\Delta_r H^\circ}{RT^2} < 0$ car exothermique

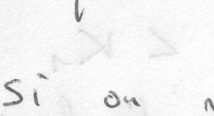
si $T \nearrow$ $dT > 0$ et $d\ln K^\circ < 0$ → $\underline{K^\circ \searrow}$

équilibre déplacé ds le sens ← indirect.

6) Le Chatelier : si on $\nearrow P$, l'équilibre évolue ds le sens qui détruit du gaz => ici ds le sens direct.

7) si on veut obtenir ZrO_2, alors on a intérêt à $\nearrow P$ et $\searrow T$. Mais Δ à la cinétique

II Synthèse de l'ammoniac

1) 

2) On a $8 \times 1/8 + 1 = 2$ atomes de Fer par maille

et $\rho = \dfrac{2 \times m}{a^3}$ et $m = \dfrac{M}{\mathcal{N}_a} = \dfrac{112\ 10^{-3}}{\mathcal{N}_a}$

$= \dfrac{11\ 10^{-1}}{6\,10^{23} \times (3\,10^{-10})^3} = \dfrac{11}{6 \times 27} \times 10^5 = 6{,}7\,10^3\ kg/m^3$

3) $\Delta_r H^o_1 < 0$ → exothermique → dégagement d'énergie, favorisé à ~~bas~~ si $T \searrow$

$\Delta_r S^o_1 < 0$ logique car on détruit du gaz.

4) $K_1 = \exp\left(-\dfrac{\Delta_r G^o}{RT}\right)$ et $\Delta_r G^o = \Delta_r H^o - T\Delta_r S^o$

5)

	$3H_{2(g)}$ +	$N_{2(g)}$	⇄ $2NH_3(g)$
ei	$3n$	n	0
eF	$3n - 3\xi$	$n - \xi$	2ξ

si la réaction est totale, $\underline{\xi = n}$

$r = \dfrac{\xi}{n}$

$$K_1 = \frac{P_{NH_3}^2\ P_0^2}{P_{H_2}^3\ P_{N_2}} = \frac{n_{NH_3}^2\ n_{tot}^2\ P_0^2}{n_{H_2}^3\ n_{N_2}\ P^2}$$

or $n_{NH_3} = 2\xi = 2rn$; $n_{H_2} = 3(n-\xi) = 3n(1-r)$

$n_{N_2} = n - \xi = n(1-r)$ et $n_{tot} = 4n - 2\xi = 2n(2-r)$

$$K_1 = \frac{4r^2n^2 \times 4n^2(2-r)^2}{27n^3(1-r)^3 \times n(1-r)}\ \frac{P_0^2}{P^2}$$

$$= \frac{16\,r^2(2-r)^2}{27(1-r)^4}\ \frac{P_0^2}{P^2}$$

6) ~~Le~~ Chatelier où ici si $P \nearrow$ alors $Q < K_1$

$\Delta_r G = RT \ln \dfrac{Q}{K_1} < 0$ → déplacement dans le sens direct.

du coup on prend P très élevé → 300 bar >> P_0

7) Van't Hoff $\Delta_r H^o_1 < 0$ → si $T \nearrow$ déplacement dans le sens indirect.

8) du coup 25°C paraît plus adapté que 450°C

Mais il y a la cinétique qui ⇒ T ↗

Catalyseur → ↗ la cinétique sans qu'on ait besoin ↗ T !

III Ondes acoustiques

1) $dm(t) = \mu(x,t) \times A(x,t)\, dx$ $\quad (dm = \mu\, d\tau)$

$dm(t+dt) = \mu(x, t+dt) \times A(x, t+dt)\, dx.$

2) $\delta m_e = D_{me}\, dt$ or $D_{me} = \mu(x,t)\, v(x,t)\, A(x,t)$

$\delta m_s = D_{ms}\, dt$ " $D_{ms} = \mu(x+dx,t)\, v(x+dx,t)\, A(x+dx,t)$

3) $dm(t+dt) - dm(t) = \delta m_e - \delta m_s$ $\quad \left(\frac{dm}{dt} = - D_m^{\text{entrant}} \right)$

$$\hookrightarrow \frac{\partial}{\partial t}(\mu A)\, dt\, dx = -\frac{\partial}{\partial x}(\mu v A)\, dx\, dt$$

$$\hookrightarrow \frac{\partial}{\partial t}(\mu A) + \frac{\partial}{\partial x}(\mu A v) = 0$$

4) $$\frac{\partial}{\partial t}\left[(\mu_0+\mu_1)(A_0+a_1)\right] + \frac{\partial}{\partial x}\left[(\mu_0+\mu_1)(A_0+a_1)\vec{v}\right] = 0$$

$\mu_0 A_0 = \text{cte} \rightarrow \frac{\partial}{\partial t}$ et $\frac{\partial}{\partial x} = 0$ $\qquad \mu_1 a_1,\ \mu_1 v,\ a_1 v \rightarrow 2^{\circ}$ ordre négligé

$$\frac{\partial}{\partial t}(\mu_1 A_0 + \mu_0 a_1) + \frac{\partial}{\partial x}(\mu_0 A_0 v) = 0$$

$$A_0 \frac{\partial \mu_1}{\partial t} + \mu_0 \frac{\partial a_1}{\partial t} + \mu_0 A_0 \frac{\partial v}{\partial x} = 0$$

5) RFD à $d\tau$ → $\mu\, d\tau \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot \vec{\text{grad}})\vec{v} \right) = -\vec{\text{grad}}\, P\, d\tau$

terme local $\qquad$ terme convectif.

6) $(\vec{v}\cdot\vec{\text{grad}})\vec{v} \rightarrow 2^{\circ}$ ordre

$$\mu_0 \frac{\partial \vec{v}}{\partial t} = -\vec{\text{grad}}\, p_1$$

$$\hookrightarrow \mu_0 \frac{\partial v}{\partial t} = -\frac{\partial p_1}{\partial x}$$

7) $\left(\frac{\partial A}{\partial P}\right) = \frac{a_1}{p_1} \rightarrow D A p_1 = a_1$ au 1° ordre $\rightarrow D A_0 p_1 = a_1$

8) $\chi_s = \frac{1}{\mu}\left(\frac{\partial \mu}{\partial P}\right) = \frac{1}{\mu}\frac{\mu_1}{p_1} \rightarrow \chi_s \mu p_1 = \mu_1$

$\rightarrow \mu_1 = \chi_s p_1 \mu_0$) au 1° ordre

9) $\mu_0 \frac{\partial a_1}{\partial t} + A_0 \frac{\partial \mu_1}{\partial t} + \mu_0 A_0 \frac{\partial v}{\partial x} = \mu_0 D A_0 \frac{\partial p_1}{\partial t} + \chi_s \mu_0 A_0 \frac{\partial p_1}{\partial t} + \mu_0 A_0 \frac{\partial v}{\partial x}$

d'où $(D+\chi_s)\frac{\partial p_1}{\partial t} + \frac{\partial v}{\partial x} = 0$

Schwarz

10) $(D+\chi_s)\frac{\partial^2 p_1}{\partial t^2} = -\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}\right) = -\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial t}\right) = +\frac{1}{\mu_0}\frac{\partial^2 p_1}{\partial x^2}$

$\frac{\partial^2 p_1}{\partial x^2} - (D+\chi_s)\mu_0 \frac{\partial^2 p_1}{\partial t^2} = 0 \qquad 1/c^2 = (D+\chi_s)\mu_0$

d'où $c = \frac{1}{\sqrt{\mu_0 (D+\chi_s)}}$ D et χ_s sont en Pa^{-1} $= N^{-1} m^2$ $= kg^{-1} m\, s^2$

$\mu_0 (D+\chi_s) \rightarrow kg\, m^{-3} . (kg^{-1} m\, s^{+2}) = m^{-2} s^2$

$= 1/c^2$

11) $c = \frac{1}{\sqrt{1{,}3 (12\; 10^{-6})}} \simeq \frac{1}{4} 10^3 = 250$ m/s

IV Echolocalisation du dauphin

1) Loi de Laplace $PV^\gamma = cte$ appliqué au volume $V + Sx$

$P = P_{int} = P_{ext} + p \qquad (P_{ext} + p)(V+Sx)^\gamma = cte$) à $t=0$

car à $t=0$ $p(\xi=0)=0$ et le volume $= V + Sx$

à t on a $p(\xi) = p$ et le volume $= V + Sx + \xi S$

$P_{ext}(V+Sx)^\gamma = (P_{ext}+p)(V+Sx+S\xi)^\gamma$

$= P_{ext}(1+p/P_{ext})(V+Sx)^\gamma \left(1 + \frac{S\xi}{V+Sx}\right)^\gamma$

$1 = (1 + p/P_{ext})\left(1 + \frac{\gamma S \xi}{V+Sx}\right)$

$$0 = \frac{p}{P_{ext}} + \frac{\gamma s \xi}{V + S x} \rightarrow p(\xi) = -\frac{P_{ext} S \gamma \xi}{V + Sx} \quad \text{or } Sx \ll V$$

$$\simeq -\frac{P_{ext} S \gamma}{V} \xi$$

RFD à l'élément de fluide $\quad dm\,\vec{a} = \rho S dx \frac{\partial^2 \xi}{\partial t^2} \vec{e_x}$

$$= (P_{int} - P_{ext}) S \vec{e_x}$$

$$\rho S dx \frac{\partial^2 \xi}{\partial t^2} = -\frac{P_{ext} S^2 \gamma}{V} \xi \quad \text{car } P_{int} = P_{ext} + p$$

$$\frac{\partial^2 \xi}{\partial t^2} + \frac{P_{ext} S \gamma}{\rho V dx} \xi = 0 \rightarrow \text{oscillateur harmonique}$$

$$\omega_0 = \sqrt{\frac{P_{ext} S \gamma}{\rho V dx}}$$

2) $I(50m) = \frac{I(r=0)}{2} = I(r=0)\, e^{-\alpha \times 50} \rightarrow e^{\alpha 50} = 2$

$\rightarrow \alpha 50 = \ln 2$

$\rightarrow \alpha = \frac{1}{50} \ln 2 = 1{,}4\ 10^{-2}\ m^{-1}$

$$R = \frac{(60)^2}{(3020)^2} = \left(\frac{60}{3\ 10^3}\right)^2 = 4 \times 10^{-4}$$

$I(r=0) \xrightarrow{75m} I(r=75m)$, il se réfléchi $R\, I(r=75m)$

puis on a à nouveau l'amortissement $\quad I_{Final} = I(0)\, R\, e^{-2\alpha \times 75}$

$$\mathcal{R} = R\, e^{-150\alpha} = R \left(\frac{1}{2}\right)^3 = \frac{R}{8} = 0{,}5\ 10^{-4} = 5\ 10^{-5}$$

retour faible !

sur la $\simeq$ ligne $\theta = 0 \rightarrow \quad U = \frac{\delta f c}{2F} = \frac{10^3 \times 1{,}4\ 10^3}{2 \times 125\ 10^3} = \frac{1{,}4\ 10^3}{250}$

$= 4 \times 1{,}4$

soit $U = 20\ km/h$ $\quad = 5{,}6\ m/s$

V Ondes sonores

1) cf cours $\quad p_1 \simeq m\,Pa$; $v \simeq 10^{-5}\ m/s$ et $\rho_1 = \rho_0 \overbrace{\chi_s}^{10^{-5}} p_1$

$\simeq 10^{-8}\ kg/m^3$

$F \in [10\,Hz, 10\,kHz]$

2) $\chi_S = \frac{1}{\mu}\left(\frac{\partial \mu}{\partial P}\right)_S \rightarrow \mu_0 \chi_S p_1 = \mu_1$ (cf cours)

air subit un écoulement parfait → isentropique.

χ_S → compressibilité $\chi_S = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S$ → variation de

V en fonction de P, incompressible → $\chi_S = 0$

3) Euler → $\mu_0 \frac{\partial v}{\partial t} = -\frac{\partial p_1}{\partial x}$ et $\frac{\partial \mu}{\partial t} = -\text{div}\,\mu \vec{v}$

$\frac{\partial \mu_1}{\partial t} = -\mu_0 \frac{\partial v}{\partial x} \rightarrow \mu_0 \chi_S \frac{\partial p_1}{\partial t} = -\mu_0 \frac{\partial v}{\partial x} \rightarrow -\frac{\partial v}{\partial x}\frac{1}{\chi_S} = \frac{\partial p_1}{\partial t}$

or $\mu_0 \frac{\partial v}{\partial t} = -\frac{\partial p_1}{\partial x}$ → $-\frac{\partial^2 p_1}{\partial x^2} = \mu_0 \frac{\partial}{\partial x}\left(\frac{\partial v}{\partial t}\right) = \mu_0 \frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}\right)$
↳ Schwarz

$-\frac{\partial^2 p_1}{\partial x^2} = -\mu_0 \chi_S \frac{\partial^2 p_1}{\partial t^2} \rightarrow \frac{\partial^2 p_1}{\partial x^2} + \mu_0 \chi_S \frac{\partial^2 p_1}{\partial t^2} = 0$

$1/c^2 = \mu_0 \chi_S$

4) $-\frac{\partial p_i}{\partial x} = \mu_0 \frac{\partial v_i}{\partial t} \rightarrow +jk\,\underline{p_i} = \mu_0 \frac{\underline{p_i}}{Z_a}(j\omega)$

$Z_a = \frac{\mu_0 \omega}{k}$ or $k = \frac{\omega}{c}$ → D'Alembert $Z_a = 340$ SI

$= c\mu_0 = \sqrt{\frac{\mu_0}{\chi_S}}$ → impédance acoustique

$\underline{v_r}(x,t) = -\frac{\underline{p_{rm}}}{Z_a} e^{j(\omega t + kx)}$ et $\underline{v_t} = \frac{\underline{p_{tm}}}{Z_a} e^{j(\omega t - kx)} = \frac{\underline{p_t}}{Z_a}$

$= -\frac{\underline{p_r}}{Z_a}$

5) Continuité de la vitesse $v(0^-) = v(0^+)$

$\underline{v_{im}} + \underline{v_{rm}} = \underline{v_{tm}}$

le déplacement de la cloison est très faible

$v \simeq \frac{\Delta x}{T} \simeq F\Delta x \rightarrow \Delta x \simeq \frac{v}{F} = \frac{10^{-5}}{10^3} \simeq 10^{-8}\,m$

6) $m\vec{a} = -K\,X(t)\,\vec{e_x} + \underbrace{P(x=0^-)}_{(p_i+p_r)(x=0^-)} S\,\vec{e_x} - \underbrace{P(x=0^+)}_{p_t\,(x=0^+)} S\,\vec{e_x}$

$$m\ddot{\underline{X}}_m = m(j\omega)^2 \underline{X}_m = -K\underline{X} + (\underline{P}_{mi} + \underline{P}_{-r} - \underline{P}_{-t})Se^{j\omega t}$$

$$\left(\frac{-m\omega^2+K}{S}\right)\underline{X}_m = \underline{P}_{mi} + \underline{P}_{-r} - \underline{P}_{-t}$$

et $\underline{v}_{i-} + \underline{v}_{r-} = \underline{v}_{t-} \Rightarrow \frac{P_{i-}}{Z_a} - \frac{P_{r-}}{Z_a} = \frac{P_{t-}}{Z_a}$

$$\underline{P}_{i-} - \underline{P}_{r-} = \underline{P}_{t-} \rightarrow \underline{P}_{r-} = \underline{P}_{i-} - \underline{P}_{t-}$$

$$\left(\frac{-m\omega^2+K}{S}\right)\underline{X}_m = 2\underline{P}_{mi} - 2\underline{P}_{t-}$$

De plus $v(0^+) = \dot{X} = \underline{X}_- j\omega e^{j\omega t} = \underline{v}_{t-} e^{j\omega t}$

$$= \frac{\underline{P}_{t-}}{Z_a} e^{j\omega t}$$

$$\underline{X}_m = \frac{\underline{P}_{t-}}{j\omega Z_a}$$

$$\left(\left(\frac{-m\omega^2+K}{S}\right)\frac{1}{j\omega Z_a} + 2\right)\underline{P}_{t-} = 2\underline{P}_{-i}$$

$$\frac{\underline{P}_{t-}}{\underline{P}_{ti}} = \frac{2}{2 + \frac{j\omega m}{Z_a S} + \frac{K}{Sj\omega Z_a}} = \frac{1}{1 + \frac{1}{2Z_a S}\left(j\omega m + \frac{K}{j\omega}\right)}$$

d'où $H_0 = 1$ et $\frac{Q}{\omega_0} = \frac{m}{2Z_a S}$ et $Q\omega_0 = \frac{K}{2Z_a S}$

$$Q^2 = \frac{mK}{4Z_a^2 S^2} \rightarrow Q = \frac{1}{2Z_a S}\sqrt{mK} \text{ et } \omega_0 = \sqrt{\frac{K}{m}}$$

On reconnait un passe-bande. $\omega = \omega_0 \rightarrow$ résonance.

7) On veut $m\omega^2 X \gg KX \rightarrow \omega^2 \gg \frac{K}{m} = \omega_0^2$

$$\underline{H}(j\omega) \simeq \frac{H_0}{1 + jQ\frac{\omega}{\omega_0}} \rightarrow \text{passe-bas}$$

$$10\log\left(\frac{\langle \Pi_t \rangle}{\langle \Pi_i \rangle}\right) = -40\text{ dB} = 20\log\left(\left|\frac{P_i}{P_t}\right|\right) \qquad \frac{Q^2\omega^2}{\omega_0^2} \gg 1$$

$$-20 = \log|H| = \log\frac{1}{\sqrt{1+Q^2\frac{\omega^2}{\omega_0^2}}} \simeq -1\log\frac{Q\omega}{\omega_0}$$

$$\frac{\Omega\omega}{\omega_0} = \cancel{0{,}011}\ 100 \quad \text{or} \quad \frac{\Omega}{\omega_0} = \frac{m}{2Z_0 S} = \frac{N c S e}{2 Z_a S} = \frac{N c e}{2 Z_a} \quad \text{et } Z_a =$$

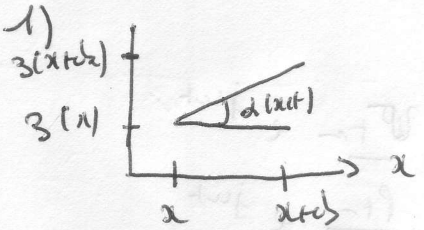
$$e = \frac{\cancel{0{,}011}\ 100 \times 2 \times Z_a}{N c \times 2\pi F} = \frac{\cancel{100} \times 340}{1{,}1\cdot 10^3 \times 3 \times 200} = \frac{3{,}4}{1{,}4} \cancel{10^{-3}}$$

$$= \frac{3{,}4}{7{,}2} 10^{-1} = 5\ 10^{-1}\ m$$

$$\simeq 5\ cm$$

VII Production du son

1)



$$\sin\alpha \simeq \alpha = \frac{z(x+dx) - z(x)}{ds} = \frac{1}{ds} \frac{\partial z}{\partial x} dx$$

$$\text{or} \quad ds = \sqrt{dx^2 + \left(\frac{\partial z}{\partial x}\right)^2 dx^2} \quad \rightarrow \text{pythagore}$$

$$= dx \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2}$$

$$\sin\alpha = \frac{\partial z / \partial x}{\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2}} \simeq \frac{\partial z}{\partial x}$$

$$\cos\alpha = \frac{dx}{ds} = \frac{1}{\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2}} \simeq 1$$

$$dm\,\vec{a} = \mu\, ds\, \vec{a} = \mu\, dx \frac{\partial^2 z}{\partial t^2} \vec{e_z} = \vec{T}(x+dx) - \vec{T}(x)$$

$$\text{sur } \vec{e_x} \rightarrow 0 = T(x+dx)\cos\alpha(x+dx) - T(x)\cos\alpha(x)$$

$$\frac{\partial T}{\partial x} = 0 \quad \text{ou} \quad \text{plutôt ici} \quad 0 = T_x(x+dx) - T_x(x)$$

$$\hookrightarrow \frac{\partial T_x}{\partial x} = 0 \rightarrow T_x = \text{cte sur } x$$

$$\text{or} \quad T_x = T\cos\alpha \simeq T = \text{cte sur } x$$

2) cf cours on a $c^2 = \frac{T}{\mu} \rightarrow c = \sqrt{\frac{T}{\mu}}$ vitesse de propagation.

3) Montrer ?! $\frac{\partial^2 f}{\partial t^2} = (-c)^2 \frac{d^2 f}{du^2}$ et $\frac{\partial^2 f}{\partial x^2} = (1)^2 \times \frac{d^2 f}{du^2}$

on pose $u = x - ct$

$\hookrightarrow$ ça marche

idem avec $v = x + ct$ et $g(v)$

$f(x-ct)$ onde qui se propage ds le sns de $x \nearrow$

$g(x+ct)$ " " " $x \searrow$

4) Le fil est accroché à ses 2 extrémités

$z(0,t) = 0 = z(l,t)$

$\underline{A} + \underline{B} = 0$ et $\underline{A}\, e^{-jkl} + \underline{B}\, e^{jkl} = 0$

$\hookrightarrow A = -\underline{B}$ $\rightarrow B(\sin kl)2j = 0$ $\rightarrow \sin kl = 0$

$\underline{kl = n\pi}$

or $k = \frac{\omega}{c}$ $\rightarrow \omega_n = \frac{n\pi c}{l}$

$A = -B \Rightarrow \underline{z}(x,t) = \underline{B}\, e^{j\omega t}(-e^{-jkx} + e^{jkx})$

$= \underline{B}\, e^{j\omega t}\, 2j \sin kx$

en réel $z(x,t) = -2|B| \sin(\omega t + \varphi) \sin kx$

onde stationnaire. Vibration sans propagation

5°)

$\omega_1 = \frac{\pi c}{l}$ $\rightarrow f_1 = \frac{\omega_1}{2\pi} = \frac{c}{2l}$ $\rightarrow$ Fréquence du fondamental

$c = \sqrt{\frac{T}{\mu}} = 2lf_1$ $\rightarrow T = \mu(2lf_1)^2$

$= 5\,10^{-7}(2 \cdot 0{,}66 \times 3\,10^2)^2$

$= 5\,10^{-7}(4\,10^4) = 2\,10^{-2}\,N$

$T \ll T_e \rightarrow$ zone élastique !

6)

$$\mu(-\omega^2) - T(-k^2) + \frac{E\pi a^4}{4}\underbrace{(-jk)^4}_{k^4} = 0$$

$$k^4\left(\frac{E\pi a^4}{4}\right) + k^2 T - \omega^2\mu = 0$$

or mode propre $\Rightarrow kl = n\pi$ $\rightarrow k = \frac{n\pi}{l}$

$$\left(\frac{n\pi}{l}\right)^4\left(\frac{E\pi a^4}{4}\right) + \left(\frac{n\pi}{l}\right)^2 T = \omega^2\mu$$

$$\omega = \sqrt{\frac{1}{\mu}\left(T + \left(\frac{n\pi}{l}\right)^2\left(\frac{E\pi a^4}{4}\right)\right)}\ \frac{n\pi}{l}$$

or $\omega = 2\pi f \rightarrow f = \frac{n}{2l}\sqrt{\frac{T}{\mu}}\sqrt{1 + n^2\frac{\pi^3 E a^4}{l^2 4 T^2}}$

$$= \frac{nc}{2l}\sqrt{1+n^2\underbrace{\frac{\pi^3 E a^4}{l^2 4 T^2}}_{\beta}}$$