Equivalents usuels au voisinage de 0.

$e^x - 1 \sim x$ $\quad$ $\ln(1+x) \sim x$ $\quad$ $\forall \alpha \in \mathbb{R},\ (1+x)^\alpha - 1 \sim \alpha x$

$\sin(x) \sim x$ $\quad$ $\tan(x) \sim x$ $\quad$ $\cos(x) - 1 \sim -\dfrac{x^2}{2}$

$\text{ch}(x) - 1 \sim \dfrac{x^2}{2}$ $\quad$ $\text{sh}(x) \sim x$ $\quad$ $\arctan(x) \sim x$ $\quad$ $\arcsin(x) \sim x$

Développements limités usuels.

Au voisinage de 0 :

$$e^x = \sum_{k=0}^{n} \frac{x^k}{k!} + o(x^n) = 1 + x + \frac{x^2}{2!} + ... + \frac{x^n}{n!} + o(x^n)$$

$$\sin(x) = \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!} + o(x^{2n+2}) = x - \frac{x^3}{3!} + ... + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$$

$$\cos(x) = \sum_{k=0}^{n} \frac{(-1)^k x^{2k}}{(2k)!} + o(x^{2n+1}) = 1 - \frac{x^2}{2!} + ... + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$$

$$\text{sh}x = \sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2}) = x + \frac{x^3}{3!} + ... + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$$

$$\text{ch}x = \sum_{k=0}^{n} \frac{x^{2k}}{(2k)!} + o(x^{2n+1}) = 1 + \frac{x^2}{2!} + ... + \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$$

$$\ln(1+x) = \sum_{k=1}^{n} \frac{(-1)^{k-1} x^k}{k} + o(x^n) = x - \frac{x^2}{2} + \frac{x^3}{3} + ... + (-1)^{n-1} \frac{x^n}{n} + o(x^n)$$

$$\frac{1}{1+x} = \sum_{k=0}^{n} (-1)^k x^k + o(x^n) = 1 - x + x^2 + ... + (-1)^n x^n + o(x^n)$$

$$\frac{1}{1-x} = \sum_{k=0}^{n} x^k + o(x^n) = 1 + x + x^2 + ... + x^n + o(x^n)$$

$$\forall \alpha \in \mathbb{R},\ (1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + ... + \frac{\alpha(\alpha-1)...(\alpha-n+1)}{n!} x^n + o(x^n)$$

$$\arctan(x) = \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{2k+1} + o(x^{2n+2}) = x - \frac{x^3}{3} + ... + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})$$

En particulier pour $\alpha = \dfrac{1}{2}$ ou $-\dfrac{1}{2}$ (et n=3) :

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + o(x^3) \text{ et } \frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + o(x^3)$$

Autre résultat au programme : $\tan(x) = x + \dfrac{x^3}{3} + o(x^4)$