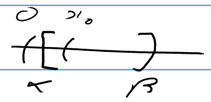
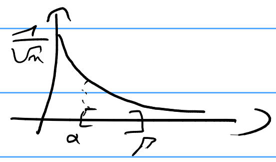
$$f = \sum_{n=0}^{\infty} f_n \quad \mathcal{C}^0(]0;+\infty[) \ ?$$



$$\boxed{\forall \alpha, \beta, \text{ CVU sur } [\alpha,\beta] \subset]0;+\infty[} \Longrightarrow \boxed{\forall \alpha, \beta, \ f \in \mathcal{C}^0([\alpha,\beta])} \Longrightarrow \boxed{f \in \mathcal{C}^0(]0;+\infty[)}$$

$$\boxed{\text{CVU sur }]0;+\infty[}$$



$$f(x) = \sum_{n=1}^{\infty} \underbrace{\frac{e^{-nx}}{\sqrt{n}}}_{f_n(x)} \quad \text{pour } x \in]0,+\infty[$$

$$\|f_n\|_\infty = \frac{1}{\sqrt{n}}$$

$\cancel{\text{CVU}}$

- Si $[\alpha,\beta] \subset]0,+\infty[$

$$\|f_n\|_{\infty,[\alpha,\beta]} = f_n(\alpha)$$

$\sum f_n(\alpha)$ CV donc CVN sur $[\alpha,\beta]$

donc CVU. Chaque f_n est $\mathcal{C}^0$

donc f est $\mathcal{C}([\alpha,\beta])$

CVU sur chaque $[\alpha,\beta]$

Ceci est vrai $\forall [\alpha,\beta] \subset]0,+\infty[$, donc

$$\boxed{f \in \mathcal{C}(]0,+\infty[)}$$

- Il n'y a pas CVN

[Fait : pas de CVU sur $]0,+\infty[$

1) Th de la double limite

$]0,+\infty[$

$$\left.\begin{array}{l} f_n(x) \to \alpha_n \\ \sum f_n \text{ CVU} \end{array}\right\} \Longrightarrow \left[\begin{array}{l} \sum \alpha_n \text{ CV} \\ \sum_{n=0}^{\infty} f_n(x) \xrightarrow[x \to 0]{} \sum_{n=0}^{\infty} \alpha_n \end{array}\right.$$

" $\lim \sum = \sum \lim$ "

$$f(x) = \sum_{0}^{\infty} \frac{e^{-nx}}{\sqrt{n}}$$

$$f_n(x) \xrightarrow[0]{} \frac{1}{\sqrt{n}}$$

$\sum \frac{1}{\sqrt{n}}$ DV

dc $\sum f_n$ $\cancel{\text{CVU}}$

2) $R_n(x) = \sum_{k=n+1}^{\infty} \frac{e^{-kx}}{\sqrt{k}}$ $\quad \|R_n\|_\infty \xrightarrow{?} 0$ $\quad R_n\left(\frac{1}{n}\right) \geq \frac{e^{-1}}{\sqrt{n}} \sqrt{n}$