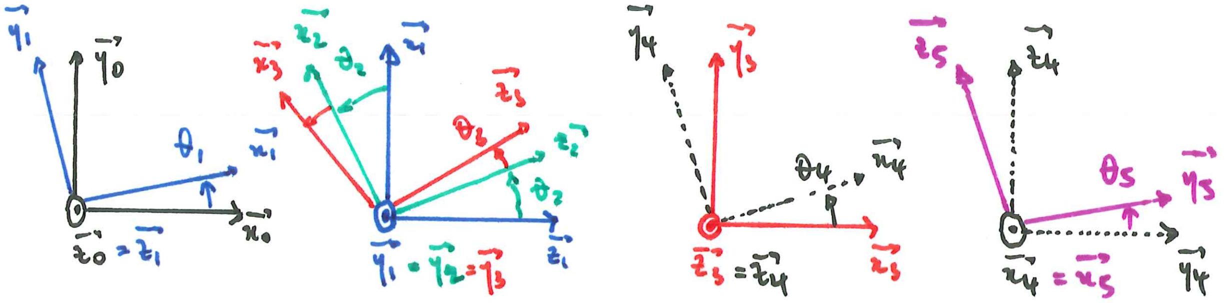
# Dance avec les robots

① $\boxed{C}$

② $$\{\mathcal{V}_{5/3}\} = \left\{ \begin{array}{l} \vec{\Omega}_{5/3} = \omega_x \cdot \vec{x}_{45} + \omega_z \cdot \vec{z}_4 \\ \vec{V}_{C\in 5/3} = \vec{0} \end{array} \right._C$$ $\boxed{D}$

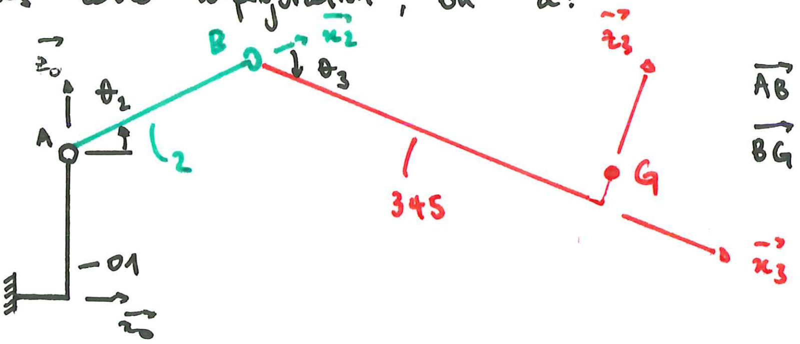
③ $$\vec{OG}\cdot\vec{x}_0 = x = \left(\vec{O_0A} + \vec{AB} + \vec{BC} + \vec{CD} + \underbrace{\vec{DG}}_{f\cdot\vec{x}_4 + h\cdot\vec{z}_5}\right)\cdot\vec{x}_0$$



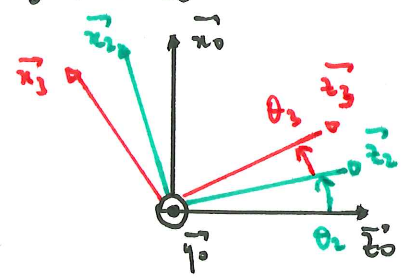

$$\vec{x}_4\cdot\vec{x}_0 = (\cos\theta_4\cdot\vec{x}_3 + \sin\theta_4\cdot\vec{y}_{31})\cdot\vec{x}_0$$
$$= \cos\theta_4\cdot\vec{x}_3\cdot\vec{x}_0 - \sin\theta_4\cdot\sin\theta_1$$

Le terme $-f\cdot\sin\theta_4\cdot\sin\theta_1$ est manquant dans toutes les expressions fournies : $\boxed{E}$.

④ Dans cette configuration, on a :




$$\vec{AB} = c\cdot\vec{x}_2$$
$$\vec{BG} = d\cdot\vec{x}_3 + e\cdot\vec{x}_{43} + f\cdot\vec{x}_{45} + h\cdot\vec{z}_{543} = X\cdot\vec{x}_3 + Z\cdot\vec{z}_3$$




$$\vec{V}_{G\in 5/0} = \vec{V}_{G\in 5/2} + \vec{V}_{G\in 2/0}$$

où $$\vec{V}_{G\in 5/2} = \cancel{\vec{V}_{B\in 5/2}} + \vec{GB}\wedge(\dot{\theta}_3\cdot\vec{y}_3)$$
$$= -(X\cdot\vec{x}_3 + Z\cdot\vec{z}_3)\wedge(\dot{\theta}_3\cdot\vec{y}_3)$$
$$= -X\cdot\dot{\theta}_3\cdot\vec{z}_3 + Z\cdot\dot{\theta}_3\cdot\vec{x}_3$$ ,

$$\vec{V}_{G\in 2/0} = \cancel{\vec{V}_{A\in 2/0}} + \vec{GA} \wedge (\dot\theta_2 \cdot \vec{y_3})$$

$$= -(X\cdot\vec{x_3} + Z\cdot\vec{z_3} + c\cdot\vec{x_2}) \wedge (\dot\theta_2\cdot\vec{y_{32}})$$

$$= -X\cdot\dot\theta_2\cdot\vec{z_3} + Z\cdot\dot\theta_2\cdot\vec{x_3} - c\cdot\dot\theta_2\cdot\vec{z_2}$$

$$\vec{V}_{G\in 5/0} = -X\cdot(\dot\theta_2+\dot\theta_3)\cdot\vec{z_3} + Z\cdot(\dot\theta_2+\dot\theta_3)\cdot\vec{x_3} - c\cdot\dot\theta_2\cdot\vec{z_2}$$

Il faut que :

$$\vec{V}_{G\in 5/0}\cdot\vec{z_0} = 0 = -X\cdot(\dot\theta_2+\dot\theta_3)\cdot\cos(\theta_2+\theta_3)$$
$$- Z\cdot(\dot\theta_2+\dot\theta_3)\cdot\sin(\theta_2+\theta_3)$$
$$- c\cdot\dot\theta_2\cdot\cos(\theta_2)$$

$\boxed{E}$

5 Et :

$$\vec{V}_{G\in 5/0}\cdot\vec{x_0} = -X\cdot(\dot\theta_2+\dot\theta_3)\cdot\sin(\theta_2+\theta_3)$$
$$+ Z\cdot(\dot\theta_2+\dot\theta_3)\cdot\cos(\theta_2+\theta_3)$$
$$- c\cdot\dot\theta_2\cdot\sin(\theta_2)$$

$\boxed{E}$

6 Calculons :

$$\vec{V}_{G\in 5/0} = \vec{V}_{G\in 5/4} + \vec{V}_{G\in 4/3} + \vec{V}_{G\in 3/2} + \vec{V}_{G\in 2/0} \quad (\text{car } \theta_1 = 0)$$

- $$\vec{V}_{G\in 5/4} = \cancel{\vec{V}_{D\in 5/4}} + \vec{GD}\wedge(\dot\theta_5\cdot\vec{x_{45}})$$

$$= -(f\cdot\vec{x_4} + h\cdot\vec{z_5})\wedge(\dot\theta_5\cdot\vec{x_{45}})$$

$$= -h\cdot\dot\theta_5\cdot\vec{y_5}$$

- $$\vec{V}_{G\in 4/3} = \cancel{\vec{V}_{C\in 4/3}} + \vec{GC}\wedge(\dot\theta_4\cdot\vec{z_{34}})$$

$$= -(f\cdot\vec{x_4} + h\cdot\vec{z_5} + e\cdot\vec{x_4})\wedge(\dot\theta_4\cdot\vec{z_{34}})$$

$$= +(e+f)\cdot\dot\theta_4\cdot\vec{y_4} + h\cdot\dot\theta_4\cdot\sin\theta_5\cdot\vec{x_{45}}$$

$$\bullet\ \vec{J}_{G\in 3/2} = \cancel{\vec{J}_{B\in 3/2}} + \vec{GB} \wedge (\dot{\theta}_3 \cdot \vec{y_{123}})$$

$$= -\left((f+e)\cdot\vec{x_4} + h\cdot\vec{z_5} + d\cdot\vec{x_3}\right) \wedge (\dot{\theta}_3\cdot\vec{y_{123}})$$

$$= -(f+e)\cdot\dot{\theta}_3\cdot\cos\theta_4\cdot\vec{z_{34}}$$
$$+ h\dot{\theta}_3\cdot\cos\theta_5\cdot\vec{x_3}$$
$$- h\dot{\theta}_3\sin\theta_5\cdot\sin\theta_4\cdot\vec{z_{34}}$$
$$- d\cdot\dot{\theta}_3\cdot\vec{z_{34}}$$

$$\| \ \vec{x_4} \wedge \vec{y_{123}} = \sin\left(-\theta_4 + \frac{\pi}{2}\right)\cdot\vec{z_{34}}$$
$$= \cos\theta_4\cdot\vec{z_{34}}$$

$$\vec{z_5} \wedge \vec{y_{123}} = \left(\cos\theta_5\cdot\vec{z_{43}} - \sin\theta_5\cdot\vec{y_4}\right) \wedge \vec{y_{123}}$$
$$= -\cos\theta_5\cdot\vec{x_3} + \sin\theta_5\cdot\sin\theta_4\cdot\vec{z_{34}}$$

$$\bullet\ \vec{J}_{G\in 2/1} = \cancel{\vec{J}_{A\in 2/1}} + \vec{GA} \wedge (\dot{\theta}_2\cdot\vec{y_{123}})$$

$$= -\left((f+e)\cdot\vec{x_4} + h\cdot\vec{z_5} + d\cdot\vec{x_3} + c\cdot\vec{x_2}\right) \wedge (\dot{\theta}_2\cdot\vec{y_{123}})$$

$$= -(f+e)\cdot\dot{\theta}_2\cdot\cos\theta_4\cdot\vec{z_{34}}$$
$$+ h\cdot\dot{\theta}_2\cdot\cos\theta_5\cdot\vec{x_3}$$
$$- h\cdot\dot{\theta}_2\cdot\sin\theta_5\cdot\sin\theta_4\cdot\vec{z_{34}}$$
$$- d\cdot\dot{\theta}_2\cdot\vec{z_{34}}$$
$$- c\cdot\dot{\theta}_2\cdot\vec{z_2}$$

$\boxed{C}$

(7) On a donc : $\vec{J}_{G\in 5/0} = -(e+f)\cdot\dot{\theta}_2\cdot\vec{z_2}$

$$- d\cdot\dot{\theta}_2\cdot\vec{z_2}$$
$$+ h\cdot\dot{\theta}_2\cdot\vec{x_2}$$
$$- c\cdot\dot{\theta}_2\cdot\vec{z_2}$$

$$= +h\cdot\dot{\theta}_2\cdot\vec{x_2} - (c+d+e+f)\cdot\dot{\theta}_2\cdot\vec{z_2}$$

Avec $\dot{\theta}_2 = \text{cte}$ et $\left.\dfrac{d\vec{x_2}}{dt}\right|_0 = -\dot{\theta}_2\cdot\vec{z_2}$

$$\left.\frac{d\vec{z_2}}{dt}\right|_0 = +\dot{\theta}_2\cdot\vec{x_2}$$

Donc : $\vec{\Gamma}_{GE/0} = -h.\dot{\theta}_2^2.\vec{z_2} - (c+d+e+f).\dot{\theta}_2^2.\vec{x_2}$

Et $\vec{z_2} = \cos\theta_2.\vec{z_{10}} + \sin\theta_2.\vec{x_{10}}$

$\vec{x_2} = -\sin\theta_2.\vec{z_{10}} + \cos\theta_2.\vec{x_{10}}$ $\boxed{B}$

8 On a : $\vec{\Gamma}_G = \vec{\Gamma}_{GE/0} - \vec{g} = -h.\dot{\theta}_2^2.\vec{z_2} - (c+d+e+f).\dot{\theta}_2^2.\vec{x_2} - g.\vec{z_0}$

Je remarque que $h << c+d+e+f$ et donc :

$$\vec{\Gamma}_G \simeq -(c+d+e+f).\dot{\theta}_2^2.\vec{x_2} - g.\vec{z_0}$$

$\|\vec{\Gamma}_G\|$ est maxi pour $\vec{x_2} = \vec{z_0}$ ($\theta_2 = -\frac{\pi}{2}$) et dans ce cas :

$$\|\vec{\Gamma}_G\| \simeq g + (c+d+e+f).\dot{\theta}_2^2 \simeq g + 4,1\times(1,45)^2 \simeq g + \overbrace{4,1\times\underbrace{(1,5)^2}_{2,25}}^{\simeq 9\ m/s^2}$$

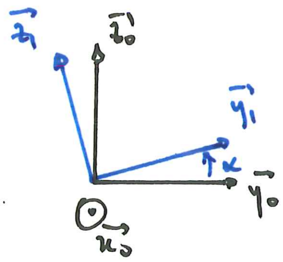
$$\simeq 2.g < 3,5.g$$ $\boxed{E}$

9 Il faudrait isoler 2 et écrire le th. des moments dynamiques en O et en projection sur $\vec{y_1}$ : $\boxed{E}$.

10 $\vec{OG_E} = \frac{1}{m_1+m_2}.(m_1.\vec{OG_1} + m_2.\vec{OG_2})$

$= \frac{m_1.l_1 + m_2.l_2}{m_1+m_2}.\vec{y_1}$ $\boxed{D}$

11 $\vec{\Omega}_{2/0} = \vec{\Omega}_{2/1} + \vec{\Omega}_{1/0} = \dot{\beta}.\vec{y_1} + \dot{\alpha}.\vec{x_1}$ $\boxed{E}$




notations de mécanique du point

12 $\vec{J}_{GE/0} = \left.\frac{d\vec{OG_E}}{dt}\right|_0 = a_E.\dot{\alpha}.\vec{z_1}$

puis $\vec{\Gamma}_{GE/0} = \left.\frac{d\vec{J}_{GE/0}}{dt}\right|_0 = a_E.\ddot{\alpha}.\vec{z_1} - a_E.\dot{\alpha}^2.\vec{y_1}$

$\boxed{B}$

13 $\boxed{D}$

(14) $I(1,O) = I(1,G_1) + I(1, G_1 \rightarrow O)$

où $\overrightarrow{OG_1} = l_1 . \vec{y_1}$

donc $I(1, G_1 \rightarrow O) = m_1 . \begin{bmatrix} l_1^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & l_1^2 \end{bmatrix}_{B_1}$

$\boxed{D}$

(15) $\vec{\sigma}_{O,1/0} = I(1,O) . \vec{\Omega}_{1/0} + m_1 . \overrightarrow{OG_1} \wedge \cancel{\vec{J}_{O \in 1/0}}$

$= I(1,O) . (\dot{\alpha} . \vec{x_1})$

$= \begin{bmatrix} A'_1 & 0 & -E'_1 \\ 0 & B'_1 & 0 \\ -E'_1 & 0 & C'_1 \end{bmatrix}_{B_1} . \begin{bmatrix} \dot{\alpha} \\ 0 \\ 0 \end{bmatrix}_{B_1}$

$= \begin{bmatrix} A'_1 . \dot{\alpha} \\ 0 \\ -E'_1 . \dot{\alpha} \end{bmatrix}_{B_1}$

$\boxed{E}$

(16) $\vec{\sigma}_{O,2/0} = \vec{\sigma}_{G_2, 2/0} + \overrightarrow{OG_2} \wedge (m_2 . \vec{J}_{G_2 \in 2/0})$

où $\vec{J}_{G_2 \in 2/0} = \cancel{\vec{J}_{G_2 \in 2/1}} + \vec{J}_{G_2 \in 1/0}$

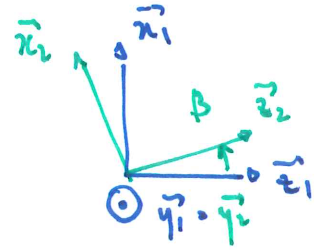
$= \cancel{\vec{J}_{O \in 1/0}} + l_2 . \vec{y_1} \wedge (\dot{\alpha} . \vec{x_1})$

$= - l_2 . \dot{\alpha} . \vec{z_1}$

donc $\overrightarrow{OG_2} \wedge (m_2 . \vec{J}_{G_2 \in 2/0}) = m_2 . l_2^2 . \dot{\alpha} . \vec{x_1}$

et $\vec{\sigma}_{G_2, 2/0} = I(2, G_2) . \vec{\Omega}_{2/0} + \vec{0}$

$$\vec{\sigma}_{G_2,2/0} = \begin{bmatrix} A_2 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & C_2 \end{bmatrix}_{B_2} \cdot \left( \dot{\beta} . \vec{y_{12}} + \dot{\alpha} . \vec{x_1} \right)$$




$$= \begin{bmatrix} A_2 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & C_2 \end{bmatrix}_{B_2} \cdot \begin{bmatrix} \dot{\alpha} . \cos\beta \\ \dot{\beta} \\ \dot{\alpha} . \sin\beta \end{bmatrix}_{B_2}$$

$$= A_2 . \dot{\alpha} . \cos\beta . \vec{x_2} + B_2 . \dot{\beta} . \vec{y_2} + C_2 . \dot{\alpha} . \sin\beta . \vec{z_2}$$

$$= A_2 . \dot{\alpha} . \cos^2\beta . \vec{x_1} - A_2 . \dot{\alpha} . \cos\beta . \sin\beta . \underline{\vec{z_1}}$$
$$+ B_2 . \dot{\beta} . \vec{y_1}$$
$$+ C_2 . \dot{\alpha} . \sin\beta . \cos\beta . \underline{\vec{z_1}} + C_2 . \dot{\alpha} . \sin^2\beta . \vec{x_1}$$

$\boxed{E}$

17 $\boxed{E}$ aucun moment cinétique ne convient.

18 • $\vec{R_d}_{E/0} = (m_1 + m_2) . \vec{\Gamma}_{G_E/0}$ → déterminé à la q° 12

$$= (m_1 + m_2) . \left( a_E . \ddot{\alpha} . \vec{z_1} - a_E . \dot{\alpha}^2 . \vec{y_1} \right)$$

• $\vec{\delta}_{0,E/0} = \vec{\delta}_{0,\{1,2\}/0}$

$$= \vec{\delta}_{0,1/0} + \vec{\delta}_{0,2/0}$$

$\boxed{B}$

19 E est soumis aux actions mécaniques extérieures suivantes :

- $0 \to E$
- Actionneur $\to E$
- Poids $\to E$

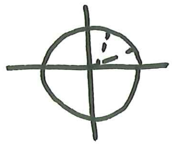
$\boxed{E}$

20 J'écris le th. des moments en O et en projection sur $\vec{x_1}$ :

$$\underbrace{\vec{M}_{0,0 \to E} . \vec{x_1}}_{0} + \underbrace{\vec{M}_{0,\text{Acti.} \to E} . \vec{x_1}}_{C_m} + \vec{M}_{0,pd \to E} . \vec{x_1} = \underbrace{\vec{\delta}_{0,E/0} . \vec{x_1}}_{\mu_0 . \dot{\alpha} . \dot{\beta} + \mu_1 . \ddot{\alpha}}$$

$$\text{et } \vec{M}_{O, pb \to E} \cdot \vec{x_1} = \cancel{\vec{M}_{G_E, pb \to E}} \cdot \vec{x_1} + \Big(\underbrace{\vec{OG_E}}_{a_E \cdot \vec{y_1}} \wedge \big(-(m_1 + m_2) \cdot g \cdot \vec{z_0}\big)\Big) \cdot \vec{x_1}$$

$$= -a_E \cdot (m_1 + m_2) \cdot g \cdot \sin\left(-\alpha + \frac{\pi}{2}\right)$$



$$= -a_E \cdot (m_1 + m_2) \cdot g \cdot \cos\alpha \quad \boxed{A}$$

21 $\boxed{A}$

22 $\boxed{A}$