Cinématique

(1) uniforme $\Rightarrow v = \text{cste} \Rightarrow \tau = \dfrac{AB+BC+CD+DE}{v} = \dfrac{425\,\text{cm}}{25\,\text{cm/s}} = \underline{17\,\text{sec}} \Rightarrow$ C

(2) circulaire uniforme $\Rightarrow v = R\dot\theta$ et $a_1 = R\dot\theta^2 = \dfrac{R^2\dot\theta^2}{R} = \dfrac{v^2}{R} = \boxed{\simeq 0{,}2\,\text{m.s}^{-2}}$ B

$BC = \dfrac{2\pi R}{4} \Rightarrow R = \dfrac{2BC}{\pi} \Rightarrow a_1 = \dfrac{(0{,}25)^2\,\cancel{\text{m}^2}\text{.s}^{-2}}{\dfrac{2\times 0{,}50}{\pi}\,\cancel{\text{m}}} = \dfrac{\pi}{16}\,\text{m.s}^{-2} \simeq \dfrac{2\times 1{,}57}{16}$

(3) $a_2 = \dfrac{v^2 \times R}{R' \times R} = a_1 \times \dfrac{R}{R'} = a_1 \times \dfrac{\cancel{2}BC/\cancel{\pi}}{\cancel{2}CD/\cancel{\pi}} = a_1 \times \dfrac{50}{75} = a_1 \times \dfrac{2\times\cancel{25}}{3\times\cancel{25}} = \dfrac{2}{3}a_1 = a_2$ C

normal virage - brusque $\Rightarrow a_1 > a_2$

(4) $\begin{cases} x_a = \text{abs. de } M_a & x = 0 \text{ en } D \\ x_b = \text{abs de } M_b & x = 2\,\text{m en } E \end{cases}$

$\left.\begin{array}{l} x_a = \underbrace{v}_{25\,\text{cm.s}^{-1}}t \\ x_b = DE - \underbrace{2v}_{50\,\text{cm.s}^{-1}}t \end{array}\right\} \to$ rencontre en $x_a = x_b$

i.e. $vt_r = DE - 2vt_r \Rightarrow t_r = \dfrac{DE}{3v} = \dfrac{200\,\text{cm}}{75\,\text{cm.s}^{-1}}$

$t_r = \dfrac{8\times\cancel{25}}{3\times\cancel{25}}\,\text{s} = 2{,}7\,\text{sec.}$ B

(5) $d_a = x_a(t_r) = vt_r = 25\,\text{cm/s} \times \dfrac{8}{3}\,\text{sec} = \dfrac{200}{3} \simeq \underline{67\,\text{cm}}$ B

(6) Soit $t_0 = t_r - \dfrac{1}{5}\,\text{sec} = \dfrac{8}{3} - \dfrac{1}{5} = \dfrac{40-3}{15} = \dfrac{37}{15}\,\text{sec.}$

~~$x_a(t_0)$~~ $d = x_b(t_0) - x_a(t_0) = DE - 2vt_0 - vt_0 = DE - 3vt_0$

$d = 200\,\text{cm} - \cancel{3}\times 25 \times \dfrac{37}{\underbrace{15}_{3\times 5}}\,\text{cm} = 200 - 5\times 37 = 200 - 185 = \underline{15\,\text{cm}}$ C

Dynamique

(7) $\not{\text{f}}$rott^s $\Rightarrow E_m = cste \Rightarrow \frac{1}{2}mv_B^2 - \underbrace{\frac{1}{2}mv_A^2}_{=0} = mg(z_A - z_B) = mg\,AB\sin\alpha$

$\Rightarrow v_B = \sqrt{2g\,\underset{L}{AB}\sin\alpha} = \sqrt{20\times 40\times \sin 30} = \sqrt{\frac{20\times 40}{2}} = \sqrt{400} = 20\,m.s^{-1}$

↳ B et C

(8) Mvt circulaire mais pas uniforme $\Rightarrow \begin{cases}\dot\theta \neq cste \\ \dot r = 0\end{cases} \Rightarrow \begin{cases}\vec v = r\dot\theta\vec e_\theta \\ \vec a = r\ddot\theta\vec e_\theta - r\dot\theta^2\vec e_r\end{cases}$

~~...~~ À cet endroit $\vec R \propto \vec P$ donc $m\vec a = \vec R + \vec P$ suivant $\vec e_z$ donc $\vec a_c \cdot \vec e_x = 0$ B

en C $\|\overrightarrow{vitesse}\|$ passe par un max donc $\dot\theta$ est max donc $\ddot\theta = 0$

donc D juste

(9) $z_B = z_0 - b\cos\alpha = b\cos\beta - b\cos\alpha \Rightarrow \mathcal{E}_p(B) = mgb(\cos\beta - \cos\alpha)$ D

(10) $E_m = cste \Rightarrow \frac{1}{2}\cancel{m}v_0^2 - \frac{1}{2}\cancel{m}v_B^2 = \cancel{mgz_B}$ ~~$\Rightarrow$~~

$\frac{1}{2}m(v_0^2 - v_B^2) = \mathcal{E}_p \Rightarrow v_0 = \sqrt{v_B^2 + \frac{2\mathcal{E}_p(B)}{m}}$ C

$v_0 < v_B$ car + haut

(11) $v_0 = v_O \leftarrow$ j'imagine faute de typo.
$\uparrow$ zéro $\uparrow$ "eau"

$a_z = -g \rightarrow v_z = -gt + v_0\sin\beta \rightarrow t_{MAX} = \frac{v_0\sin\beta}{g}$ C

↳ $z(t) = -\frac{1}{2}gt^2 + v_0\sin\beta \Rightarrow z_F = -\frac{v_0^2\sin^2\beta}{2g} + \frac{v_0^2\sin^2\beta}{g}$

$z_F = +\frac{v_0^2\sin^2\beta}{2g} = \frac{v_0^2\times\left(\frac{\sqrt{2}}{2}\right)^2}{2g} = \frac{v_0^2}{4g}$

(12) θ angle p/r à la verticale O'C

$\overrightarrow{O'M} = b\vec e_r$; $\vec v = b\dot\theta\vec e_\theta$; $\vec a = b\ddot\theta\vec e_\theta - b\dot\theta^2\vec e_r = \frac{\vec P + \vec R}{m} = +g\cos\theta\,\vec e_r - g\sin\theta\,\vec e_\theta - R\vec e_r$

↳ suivant $\vec e_\theta$: $b\ddot\theta = -g\sin\theta \simeq -g\theta$ car $\theta \ll 1$

d'où $\ddot\theta + \frac{g}{b}\theta = 0 \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{b}{g}} = 2\times 3{,}14\times\sqrt{2}$

B

$= 6{,}3\times 1{,}4$

$\times 1{,}4$; 252 ; 63 ; $8{,}82 \simeq 9$ sec C

Électrocinétique - transitoires

(13) i_{BoB} continue $\Rightarrow$ $i_L(0^+) = i_R(0^+) = 0 \Rightarrow u_R(0^+) = R\, i_R(0^+) = 0$ (A)

$\Rightarrow u_L(0^+) = E$ (D)

(14) $E = L\frac{di_L}{dt} + Ri_L \Rightarrow \frac{di_L}{dt} + \frac{R}{L}i_L = \frac{E}{L} \Rightarrow i_L = \frac{E}{R}\left[1 - \exp\left(-\frac{R}{L}t\right)\right]$ (A)

$u_L = L\frac{di_L}{dt} = \frac{EL}{R}\left(0 + \frac{R}{L}\exp\left(\frac{-R}{L}t\right)\right) = E\exp\left(\frac{-R}{L}t\right)$ (D)

(15) $i_L(0^+) = i_L(0^-) = \lim_{t\to+\infty}\left(\frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)\right) = \frac{E}{R}$ (B)

$u_R(0^+) = u_C(0) = \frac{E}{3} \Rightarrow i_R(0^+) = \frac{E}{3R}$ (D)

(16) $i_C(0^+) = i_L(0^+) - i_R(0^+) = \frac{3E}{3R} - \frac{E}{3R} = \frac{2E}{3R}$ (B)

$u_L(0^+) = E - \frac{E}{3} = \frac{2E}{3}$ (C)

(17) $E = L\frac{di_L}{dt} + Ri_R = L\frac{di_L}{dt} + R(i_L - i_C) = L\frac{di_L}{dt} + Ri_L - RC\frac{du_C}{dt}$

$E = L\frac{di_L}{dt} + Ri_L - RC\left(\frac{dE}{dt} - \frac{du_L}{dt}\right) = L\frac{di_L}{dt} + Ri_L - RC\left(0 - L\frac{d^2i_L}{dt^2}\right)$

$E = +RLC\frac{d^2i_L}{dt^2} + L\frac{di_L}{dt} + Ri_L \Leftrightarrow \frac{E}{RLC} = \frac{d^2i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{\cancel{R}1}{LC}i_L$

$\frac{E}{RLC} = \frac{d^2i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{1}{LC}i_L$

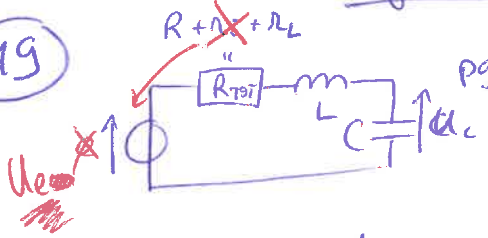
$\omega_0 = \frac{1}{\sqrt{LC}}$ (C) et $\tau_e = RC$ (A)

(18) $\frac{E}{RLC} = \omega_0^2 i_\infty = \frac{i_\infty}{LC} \Rightarrow i_\infty = \frac{E}{R}$ (B)   (D)

$t \to \infty \Rightarrow$ BoB = FiL $\Rightarrow u_C(\infty) = E \Rightarrow \mathcal{E}_C = \frac{1}{2}CE^2$

Régime sinusoïdal forcé

(19)



pdt : $\frac{\underline{u}_{c,m}}{\underline{u}_{e,m}} = \frac{\frac{1}{jC\omega}}{\frac{1}{jC\omega}+jL\omega+R+\cancel{r}+r_L} = \frac{1}{1+jR_{TOT}C\omega-LC\omega^2}$

$\frac{\underline{u}_{c,m}}{\underline{u}_{e,m}} = \frac{1}{1+j\frac{R_{TOT}}{L}\times\frac{\omega}{\omega_0^2}-\left(\frac{\omega}{\omega_0}\right)^2} = \frac{1}{1+j\frac{R_{TOT}}{L\omega_0}w-w^2}$

$1 \rightarrow H_0 = 1$ (A)

$\omega_0 = \frac{1}{\sqrt{LC}}$ (D)

$\frac{R_{TOT}}{L\omega_0} = \frac{1}{Q} \rightarrow Q = \frac{L\omega_0}{R_{TOT}} = \frac{1}{R+\cancel{r}+r_L}\sqrt{\frac{L}{C}}$

(20) $\longrightarrow$ (B)

(21) pdt

$\underline{u}_{c,m} = \underline{u}_{e,m} \times \frac{1/jC\omega}{1/jC\omega + jL\omega + R + r_L}$

$\underline{u}_{c,m} = \underline{e}_m \times \frac{1/jC\omega}{1/jC\omega + jL\omega + R + r_L + r_i}$

$\rightarrow \left(\frac{\underline{u}_{e,m}}{\underline{e}_m}\right)^{-1} = \frac{\cancel{1/jC\omega + jL\omega} + R + r_L + r_i}{\cancel{1/jC\omega + jL\omega} + R + r_L}$

Or $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$ donc

$\frac{1}{jC\omega_0} = -j\sqrt{\frac{L}{C}}$ et $jL\omega_0 = +j\sqrt{\frac{L}{C}}$

$\Rightarrow \frac{1}{jC\omega_0} = -jL\omega_0$ et

(C)

(22) $\underline{i}_m = \frac{\underline{e}_m}{\cancel{jL\omega_0 + \frac{1}{jC\omega_0}} + R + r_L + r_i} \Rightarrow$ (B)

$\underline{u}_{cm} = \frac{1}{\cancel{1} + j/Q - \cancel{1}} \times \underline{u}_{e,m} = -jQ \times \underline{u}_{em} = -jQH_0\,\underline{u}_{em}$ (C)

↑ question 19

(23) $\underline{u}_{L,m} = (jL\omega_0 + r_L)\underline{i}_m = jL\omega_0\frac{\underline{e}_m}{R+r_L+r_i} + \frac{r_L\underline{e}_m}{r_L+R+r_i}$

$\underline{u}_{L,m} = \left(\frac{r_L}{r_L+R+r_i} + j \ldots\right)\underline{e}_m \rightarrow b = \frac{1}{R+r_L+r_i}\times\sqrt{\frac{L}{C}}$

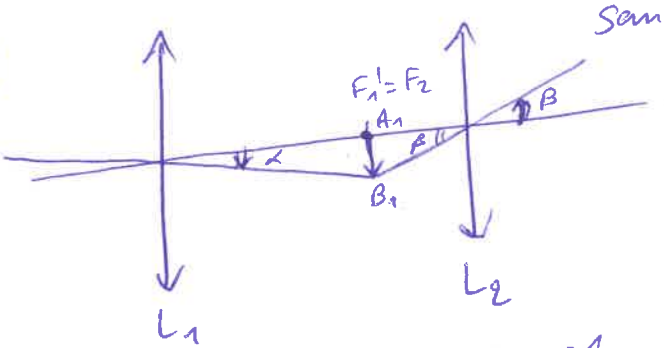
↳ a ⇒ (A)

(24) → (C)

Optique

25 (D) en accomodant

26 sans accommoder $\Rightarrow F_1' = F_2$

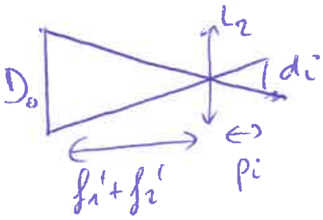


$$G_a = \frac{\beta}{\alpha} \simeq \frac{\tan\beta}{\tan\alpha} \simeq - \frac{\frac{A_1B_1}{f_2'}}{\frac{A_1B_1}{f_1'}} = -\frac{f_1'}{f_2'} \quad (B)$$

27

$$\frac{1}{p_i} + \frac{1}{f_1'+f_2'} = \frac{1}{f_2'} \Rightarrow \frac{1}{p_i} = \frac{-1}{f_1'+f_2'} + \frac{1}{f_2'} = \frac{f_1'}{f_2'(f_1'+f_2')} \Rightarrow p_i = \frac{f_2'(f_1'+f_2')}{f_1'}$$

$$p_i = f_2'\left(1+\frac{f_2'}{f_1'}\right) \quad (D)$$



$$d_i = \frac{D_0 \times p_i}{f_1'+f_2'} = D_0 \times \frac{f_2'}{f_1'} \quad (A)$$

28 On veut $\overline{O_2A'} = -d_m$ avec $\overline{O_2A_1} = \overline{O_2F_1'} = -(f_2'-x)$ donc :

image finale

$$\frac{1}{-d_m} + \frac{1}{f_2'-x} = \frac{1}{f_2'} \Rightarrow \frac{1}{f_2'-x} = \frac{d_m+f_2'}{f_2'd_m} \Rightarrow x = f_2' - \frac{f_2'd_m}{d_m+f_2'}$$

$$x = \frac{\cancel{f_2'd_m} + f_2'^2 - \cancel{f_2'd_m}}{d_m+f_2'} \Rightarrow (A)$$

29 Newton : $\begin{cases} \overline{F_1A} \times \overline{F_1'B} = -f_1'^2 \\ \overline{F_2B} \times \overline{F_2'C} = -f_2'^2 \end{cases}$ (A)

afocal $\Rightarrow F_1' = F_2$

(D) $\overline{F_1'B} \times \overline{F_2'C} = -f_2'^2$

30

$$\div \begin{pmatrix} -D_{min} \times \overline{F_1'B} = -f_1'^2 \\ -\overline{F_1'B} \times (f_2'+d_m) = -f_2'^2 \end{pmatrix} \quad (C)$$

$$D_{min} = \left(\frac{f_1'}{f_2'}\right)^2 \times (f_2'+d_m)$$

Thermodynamique

31) $\frac{V_1}{T_1} = \underbrace{\frac{nR}{P_1} = \frac{nR}{P_2}}_{\text{car } P_1 = P_2 \text{ éq. méca}} = \frac{V_2}{T_2} \Rightarrow V_2 = \frac{T_2}{T_1} V_1$; de + $V_1 + V_2 = 2V_0 = V_t$

donc $V_1\left(1 + \frac{T_2}{T_1}\right) = V_t \Rightarrow V_1 = \frac{T_1}{T_1+T_2} V_t$

éq. thermiq gaz 2 $\Rightarrow T_2 = T_0 \Rightarrow V_1 = \frac{T_1}{T_1+T_0} V_t$ (B)

$V_2 = V_t - V_1 = V_t\left(1 - \frac{T_1}{T_1+T_0}\right) = V_t\left(\frac{\cancel{T_1}+T_0-\cancel{T_1}}{T_1+T_0}\right) = \frac{T_0}{T_1+T_0} V_t$ (D)

32) (A) et $P_1 = \frac{nRT_1}{V_1} = \frac{nR\,\cancel{T_1}(T_1+T_0)}{\cancel{T_1}V_t} = \frac{nR(T_0+T_1)}{V_t}$ (B)

33) $\Delta U = \Delta U_1 + \Delta U_2 = nC_{vm}(T_1 - T_0 + \cancel{T_2 - T_0}) = \frac{nR}{\gamma-1}(T_1 - T_0)$ (B)

$\Delta U_2 = 0$ car GP et $T_i = T_f$

MAYER : $\begin{cases} C_{pm} - C_{pm} = R \\ \frac{C_{pm}}{C_{vm}} = \gamma \end{cases} \Rightarrow C_{vm}(\gamma - 1) = R$

34) $\Delta U_2 = 0 = W_2 + Q_2 \Rightarrow W_2 = -Q_2$ (D)

rév $\Rightarrow$ éq. thermique constant

$W_2 = \int_{V_0}^{V_2} -p\,dV = -\int_{V_0}^{V_2} \frac{nRT_0}{V} dV = -nRT_0 \ln\frac{V_2}{V_0} = nRT_0 \ln\frac{V_0}{V_2} = nRT_0 \ln\frac{V_t}{2V_2}$

$p = p_{ext}$ quasistatique

$W_2 = nRT_0 \ln\frac{\cancel{V_t}(T_0+T_1)}{2T_0\cancel{V_t}}$ (A)

35) W_1 reçu par gaz 1 $= -W_2 \Rightarrow \Delta U_1 = Q_1 - W_2 \Rightarrow Q_1 = \Delta U_1 + W_2$ or

on a montré que $\Delta U_2 = 0$ donc $Q_1 = \Delta U + W_2$ (B).

$P_{élec} = Ri^2 \Rightarrow Q_1 = Ri^2\tau$ (D)

36) $S_2^{(e)} = \frac{Q_2}{T_0} = \frac{-W_2}{T_0} = -nR\ln\left(\frac{T_0+T_1}{2T_0}\right) = nR\ln\left(\frac{2T_0}{T_0+T_1}\right)$ (A)

$\Delta S_2 = S_2^{(e)} + S_2^{(c)} \Rightarrow S_2^{(c)} = \Delta S_2 - S_2^{(e)} = nR\ln\left(\frac{T_0}{T_0+T_1} \times \underset{= 2}{\frac{V_t}{V_0}}\right) - nR\ln\left(\frac{2T_0}{T_0+T_1}\right)$

$S_2^{(c)} = 0$ (D) $= 0$ normal réversible.