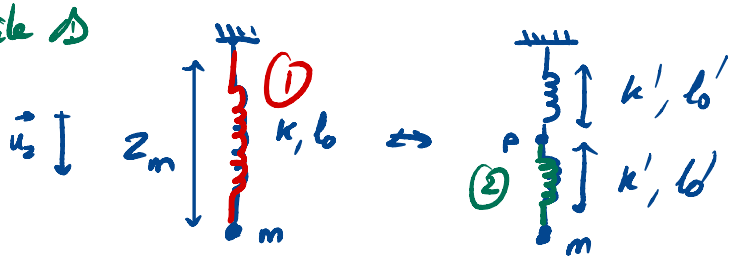
Exercice 2 : raideur d'un ressort

1 Q difficile ☹



réf : laboratoire, galiléen

à gauche : équilibre de m : $\vec{F}_{1\to m} = -k(z_m - l_0)\vec{u}_z$

$\vec{P} = mg\,\vec{u}_z$

à droite équilibre de m : $\vec{F}_{2\to m} = -k'(z_m - z_p - l_0')\vec{u}_z$

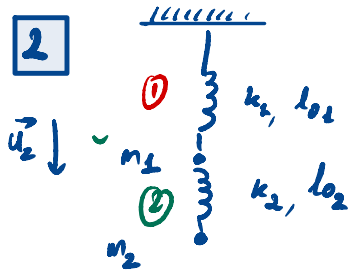
$$\text{or} \begin{cases} z_m - z_p = z_m/2 \\ l_0' = l_0/2 \end{cases}$$

$$mg = k(z_m - l_0) = k'(z_m/2 - l_0/2)$$

↑ à gauche ↑ à droite

$$\Rightarrow \boxed{k' = 2k}$$ La raideur est ×2

2



• syst m_1.

poids $\vec{P}_1 = m_1 g\,\vec{u}_z$

ressort ① $\vec{F}_{1} = -k_1 \Delta l_1\,\vec{u}_z$

ressort ② $\vec{F}_{2} = +k_2 \Delta l_2\,\vec{u}_z$

$$m_1 g = k_2 \Delta l_2 - k_2 \Delta l_2$$

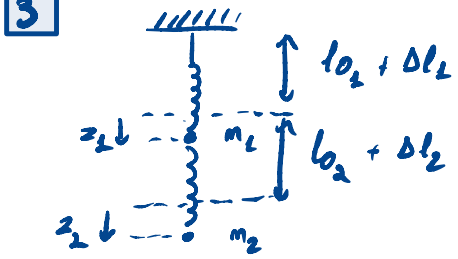
• syst m_2 Poids $\vec{P}_2 = m_2 g\,\vec{u}_z$

$\vec{F}_{2\to m_2} = -k_2 \Delta l_2\,\vec{u}_z$

$$\boxed{\Delta l_2 = m_2 g / k_2}$$

et donc $$\boxed{\Delta l_2 = \frac{m_2 g}{k_1} + \frac{m_2 g}{k_2} = (m_1 + m_2)\frac{g}{k_2}}$$

3



syst m_1 $\quad m_1 \ddot{z}_1 = m_1 g - k_1(\Delta l_1 + z_1) + k_2(\Delta l_2 + z_2 - z_1)$ PFD / $\vec{u}_z$

($-m_2 g/u_z$)

syst m_2 $\quad m_2 \ddot{z}_2 = m_2 g - k_2(\Delta l_2 + z_2 - z_1)$

$$\begin{cases} \ddot{z}_2 + \dfrac{k_2}{m_2} z_2 = \dfrac{k_2}{m_2} z_1 \\ \ddot{z}_1 + \dfrac{(k_1 + k_2)}{m_1} z_1 = \dfrac{k_2}{m_1} z_2 \end{cases}$$

4 $z_2 = 0$

$$\ddot{z}_1 + \frac{k_1 + k_2}{m_2} z_1 = 0 \Rightarrow z_1(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

$A, B \in \mathbb{R}$

et $\omega_0 = \sqrt{\frac{k_1 + k_2}{m_2}}$

$$\dot{z}_1(t) = -A\omega_0 \sin(\omega_0 t) + B\omega_0 \cos(\omega_0 t)$$

à $t = 0$, $\dot{z}_1(t=0) = 0 = B\omega_0 \Rightarrow B = 0$

$$z_1(t=0) = z_d = A$$

d'où $\boxed{z_1(t) = z_d \cos(\omega_0 t)}$

5 La Q 1 est un cas particulier de la Q 2.