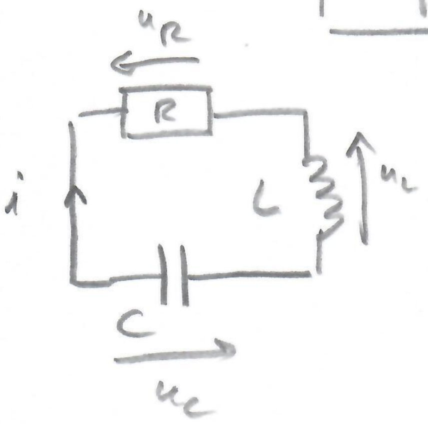
Éq° diff. 3



Loi des mailles : $u_R + u_L + u_C = 0$

$$Ri + L\frac{di}{dt} + u_C = 0$$

On $i = C\frac{du_C}{dt}$

$$LC\frac{d^2u_C}{dt^2} + RC\frac{du_C}{dt} + u_C = 0$$

On encore
$$\boxed{\ddot{u}_C + \underbrace{\frac{R}{L}}_{\frac{\omega_0}{Q} = \frac{R}{L}}\dot{u}_C + \underbrace{\frac{1}{LC}}_{\omega_0^2 = \frac{1}{LC}}u_C = 0} \quad (*)$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

$$Q = \frac{L}{\sqrt{LC}}\frac{1}{R}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

AN: $\omega_0 = \frac{1}{\sqrt{4.10^{-6}.10^{-2}}} = \frac{1}{2.10^{-4}} = \boxed{5.10^{3}\ rad.s^{-1}}$

$Q = \frac{1}{100}\sqrt{\frac{10^{-2}}{4.10^{-6}}} = \frac{1}{100}\frac{\sqrt{10^4}}{\sqrt{4}} = \boxed{\frac{1}{2}}$

Résolvons (*) :

$$r^2 + \frac{\omega_0}{Q}r + \omega_0^2 = 0$$

$$\Delta = \left(\frac{\omega_0}{Q}\right)^2 - 4\omega_0^2 = \omega_0^2\left(\frac{1}{Q^2} - 4\right) = 0$$

$\Delta = 0$ → régime critique

$$r_0 = -\frac{\omega_0}{2Q} = -\omega_0 = -\frac{1}{\tau} \qquad (2Q = 1)$$

CVacIII-1

d'r $\boxed{u_c(t) = (At + B)\, e^{-\omega_0 t}}$

À $t=0$, $u_c(t=0^+) = u_c(t=0^-) = E$

$u_c(0) = \boxed{B = E}$

Donc $u_c(t) = (At+B)\, e^{-\omega_0 t}$

$i = C\dfrac{du_c}{dt} = C\left[A - \omega_0(At+B)\right] e^{-\omega_0 t}$

$i(0) = 0$ par continuité du courant dans la bobine

$i(0) = C\left[A - \omega_0 B\right] = 0$

$\boxed{A = \omega_0 B = \omega_0 E}$

Donc au final :

$$\boxed{u_c(t) = E(1 + \omega_0 t)\, e^{-\omega_0 t}}$$