Développements usuels en 0

$\dfrac{1}{1-x} \underset{x\to 0}{=}$	$\displaystyle\sum_{k=0}^{n} x^k + \mathrm{o}(x^n) \underset{x\to 0}{=} 1 + x + x^2 + \ldots x^n + \mathrm{o}(x^n).$
$\ln(1-x) \underset{x\to 0}{=}$	$\displaystyle -\sum_{k=1}^{n} \frac{x^k}{k} + \mathrm{o}(x^n) \underset{x\to 0}{=} -x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots - \frac{x^n}{n} + \mathrm{o}(x^n).$
$\dfrac{1}{1+x} \underset{x\to 0}{=}$	$\displaystyle\sum_{k=0}^{n} (-1)^k x^k + \mathrm{o}(x^n) \underset{x\to 0}{=} 1 - x + x^2 + \ldots (-1)^n x^n + \mathrm{o}(x^n).$
$\ln(1+x) \underset{x\to 0}{=}$	$\displaystyle\sum_{k=1}^{n} (-1)^{k+1} \frac{x^k}{k} + \mathrm{o}(x^n) \underset{x\to 0}{=} x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots + (-1)^{n+1} \frac{x^n}{n} + \mathrm{o}(x^n).$
$\arctan(x) \underset{x\to 0}{=}$	$\displaystyle\sum_{k=0}^{n} (-1)^k \frac{x^{2k+1}}{2k+1} + \mathrm{o}(x^{2n+2}) \underset{x\to 0}{=} x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots + (-1)^n \frac{x^{2n+1}}{2n+1} + \mathrm{o}(x^{2n+2}).$
$(1+x)^\alpha \underset{x\to 0}{=}$	$\displaystyle 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2 + \ldots + \frac{\alpha(\alpha-1)(\alpha-2)\ldots(\alpha-n+1)}{n!} x^n + \mathrm{o}(x^n)$.
$\underset{x\to 0}{=}$	$\displaystyle\sum_{k=0}^{n} \binom{\alpha}{k}_{\mathbb{R}} x^k + \mathrm{o}(x^n)$ où $\binom{\alpha}{k}_{\mathbb{R}}$ est le coefficient du binôme généralisé à $\alpha \in \mathbb{R}$
$\sqrt{1+x} \underset{x\to 0}{=}$	$\displaystyle 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \ldots + (-1)^{n-1} \frac{1 \times 3 \times \ldots \times (2n-3)}{2 \times 4 \times \ldots \times 2n} x^n + \mathrm{o}(x^n).$
$\dfrac{1}{\sqrt{1+x}} \underset{x\to 0}{=}$	$\displaystyle 1 - \frac{1}{2}x + \frac{3}{8}x^2 + \ldots + (-1)^n \frac{1 \times 3 \times \ldots \times (2n-1)}{2 \times 4 \times \ldots \times 2n} x^n + \mathrm{o}(x^n).$
$\mathrm{e}^x \underset{x\to 0}{=}$	$\displaystyle\sum_{k=0}^{n} \frac{x^k}{k!} + \mathrm{o}(x^n) \underset{x\to 0}{=} 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots + \frac{x^n}{n!} + \mathrm{o}(x^n).$
$\cos(x) \underset{x\to 0}{=}$	$\displaystyle\sum_{k=0}^{n} \frac{(-1)^k x^{2k}}{(2k)!} + \mathrm{o}(x^{2n+1}) \underset{x\to 0}{=} 1 - \frac{x^2}{2} + \frac{x^4}{24} + \ldots + \frac{(-1)^n x^{2n}}{(2n)!} + \mathrm{o}(x^{2n+1}).$
$\sin(x) \underset{x\to 0}{=}$	$\displaystyle\sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!} + \mathrm{o}(x^{2n+2}) \underset{x\to 0}{=} x - \frac{x^3}{6} + \frac{x^5}{120} + \ldots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \mathrm{o}(x^{2n+2}).$
$\mathrm{ch}(x) \underset{x\to 0}{=}$	$\displaystyle\sum_{k=0}^{n} \frac{x^{2k}}{(2k)!} + \mathrm{o}(x^{2n+1}) \underset{x\to 0}{=} 1 + \frac{x^2}{2} + \frac{x^4}{24} + \ldots + \frac{x^{2n}}{(2n)!} + \mathrm{o}(x^{2n+1}).$
$\mathrm{sh}(x) \underset{x\to 0}{=}$	$\displaystyle\sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!} + \mathrm{o}(x^{2n+2}) \underset{x\to 0}{=} x + \frac{x^3}{6} + \frac{x^5}{120} + \ldots + \frac{x^{2n+1}}{(2n+1)!} + \mathrm{o}(x^{2n+2}).$
$\tan(x) \underset{x\to 0}{=}$	$\displaystyle x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \mathrm{o}(x^8).$

Exercice 1 : Effectuer le DL au voisinage de 0 à l'ordre indiqué de la fonction suivante :

1. $\tan^2(x)$ $\quad n = 7$
2. $\dfrac{\ln(1+x)}{1+x}$ $\quad n = 4$
3. $e^{x\sin(x)}$ $\quad n = 6$
4. $\sqrt{1+\sin(x)}$ $\quad n = 3$
5. $\sqrt{\cos(x)}$ $\quad n = 7$
6. $\dfrac{1}{\cos(x)}$ $\quad n = 5$
7. $\dfrac{x}{\sin(x)}$ $\quad n = 5$
8. $\ln\left(\dfrac{\sin(x)}{x}\right)$ $\quad n = 5$
9. $\ln\left(\dfrac{\arctan(x)}{x}\right)$ $\quad n = 7$
10. $(1+x)^x$ $\quad n = 5$
11. $(1+x)^{\frac{1}{x}}$ $\quad n = 3$
12. $\ln\left(\ln\left((1+x)^{\frac{1}{x}}\right)\right)$ $\quad n = 4$
13. $\left(\dfrac{\sin(x)}{x}\right)^{\frac{3}{x^2}}$ $\quad n = 3$
14. $(\cos(x))^p$ $\quad n = 6$
15. $\left(\dfrac{1+e^x}{2}\right)^p$ $\quad n = 2$
16. $\left(\dfrac{\sin(x)}{x}\right)^p$ $\quad n = 5$

Exercice 2 (Applications) : Calculer les limites suivantes :

1. $\displaystyle\lim_{x\to 0} \frac{\sin(\sin(x)-x)}{\sqrt{1+x^3}-1}$
2. $\displaystyle\lim_{x\to 0} \frac{3\tan 4x - 4\tan 3x}{3\sin 4x - 4\sin 3x}$
3. $\displaystyle\lim_{x\to 0} \frac{x(e^x+1)-2(e^x-1)}{x^3}$
4. $\displaystyle\lim_{x\to 0} \frac{\ln(1+x)-\ln(1-x)}{\arctan(1+x)-\arctan(1-x)}$
5. $\displaystyle\lim_{x\to 0} \left(\frac{1}{x}-\frac{1}{\ln(1+x)}\right)$
6. $\displaystyle\lim_{x\to 0} \frac{(1+x)^{\frac{1}{x}}-e}{x}$
7. $\displaystyle\lim_{x\to 1} \frac{1-x+\ln(x)}{1-\sqrt{2x-x^2}}$
8. $\displaystyle\lim_{x\to 1} \left(\frac{1}{2(1-\sqrt{x})}-\frac{1}{3(1-\sqrt[3]{x})}\right)$