Condensateur idéal de capacité C :

$i = C\frac{du}{dt} \leadsto \underline{i} = C\frac{d\underline{u}}{dt} = Cj\omega\underline{u}$

$\Rightarrow \boxed{\underline{Z}_c = \frac{\underline{u}}{\underline{i}} = \frac{1}{jC\omega}}$

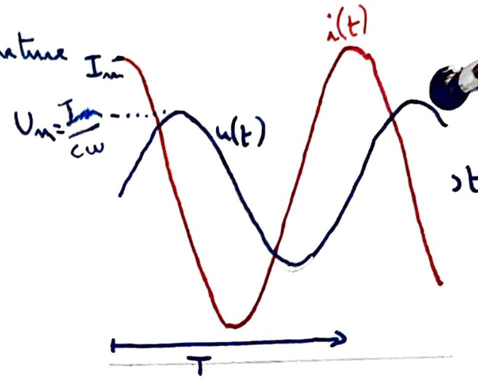
$\underline{Z}_c = \frac{1}{C\omega}e^{-j\frac{\pi}{2}}$

$\underline{Z}_c = \frac{U_m}{I_m}e^{j(\varphi_u-\varphi_i)} = \frac{1}{C\omega}e^{-j\frac{\pi}{2}}$

$|\underline{Z}_c| = \frac{U_m}{I_m} = \frac{1}{C\omega}$

$\varphi = \arg(\underline{Z}_c) = \varphi_u - \varphi_i = -\frac{\pi}{2}$

u est en retard / i (quadrature retard) $\Delta t = \frac{-T}{4}$



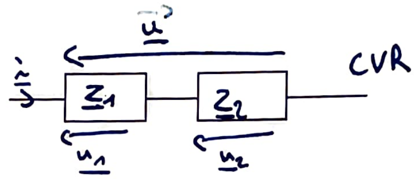
c) Association de dipôles :

- en série : $u(t) = u_1(t) + u_2(t)$

$\underline{u}(t) = \underline{u}_1(t) + \underline{u}_2(t)$

$\underline{u} = \underline{Z}_1\underline{i} + \underline{Z}_2\underline{i} = \underline{i}(\underline{Z}_1+\underline{Z}_2)$

$\underline{u} = \underline{Z}_{eq}\,\underline{i}$ où $\boxed{\underline{Z}_{eq} = \underline{Z}_1 + \underline{Z}_2}$



Pont diviseur de tension : $\frac{\underline{u}_1}{\underline{u}} = \frac{\underline{Z}_1 \cancel{\underline{i}}}{(\underline{Z}_2+\underline{Z}_1)\cancel{\underline{i}}}$

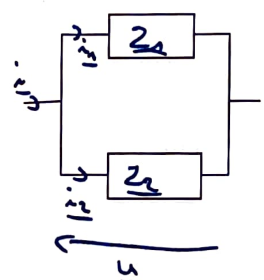
- en parallèle :

$i = i_1 + i_2 \leadsto \underline{i} = \underline{i}_1 + \underline{i}_2$

$\underline{i} = \frac{\underline{u}}{\underline{Z}_1} + \frac{\underline{u}}{\underline{Z}_2} = \underline{u}\left(\frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2}\right)$

$\underline{i} = \frac{\underline{u}}{\underline{Z}_{eq}}$ où $\boxed{\frac{1}{\underline{Z}_{eq}} = \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2}} \Leftrightarrow \boxed{\underline{Y}_{eq} = \underline{Y}_1 + \underline{Y}_2}$

$\frac{\underline{i}_1}{\underline{i}} = \frac{\frac{\underline{u}}{\underline{Z}_1}}{\frac{\underline{u}}{\underline{Z}_{eq}}} \Rightarrow \frac{\frac{1}{\underline{Z}_1}}{\frac{1}{\underline{Z}_1}+\frac{1}{\underline{Z}_2}} = \frac{\underline{i}_1}{\underline{i}} = \frac{\underline{Y}_1}{\underline{Y}_1+\underline{Y}_2}$



d) Générateur :

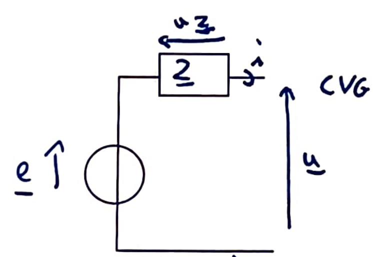
Modèle de Thévenin :

$\underline{u} = \underline{e} - \underline{u}_Z = \underline{e} - \underline{Z}\,\underline{i}$

$\Rightarrow \underline{Z}\,\underline{i} = \underline{e} - \underline{u}$

$\Rightarrow \underline{i} = \frac{\underline{e}}{\underline{Z}} - \frac{\underline{u}}{\underline{Z}}$

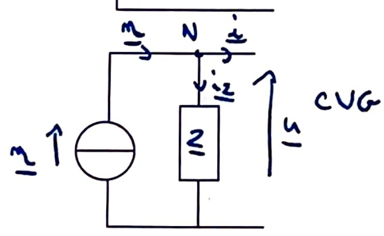
$\Rightarrow \boxed{\underline{i} = \underline{\eta} - \frac{\underline{u}}{\underline{Z}}}$ où $\underline{\eta} = \frac{\underline{e}}{\underline{Z}}$ (Loi des noeuds en N)



$\underline{e}$ fém complexe (force électromotrice)

$\underline{\eta}$ courant électromoteur complexe

Modèle de Norton.



$\underline{i}_2 = \frac{\underline{u}}{\underline{Z}}$

Remarque importante : Schéma équivalents à basse et haute fréquence.

À basses fréquences : $\boxed{f \to 0}$ $\omega = 2\pi f$ $\omega \to 0$

Bobine idéale : $|\underline{Z_L}| = \frac{U_m}{I_m} = L\omega \Rightarrow U_m = L\omega\, I_m$

$\forall I_m$, quand $\omega \to 0$ $U_m \to 0$ bobine équivalent à 1 fil

On retrouve le modèle équivalent du régime permanent continu.

Condensateur parfait : $|\underline{Z_C}| = \frac{U_m}{I_m} = \frac{1}{C\omega}$

$\Rightarrow \frac{I_m}{U_m} = C\omega \Rightarrow I_m = C\omega\, U_m$

$\forall U_m$, quand $\omega \to 0$, $I_m \to 0$

Condensateur équivalent à 1 interrupteur ouvert.

À hautes fréquences : $\omega \to +\infty$ $f \to +\infty$

Bobine idéale $|\underline{Z_L}| = \frac{U_m}{I_m} = L\omega \Rightarrow \frac{I_m}{U_m} = \frac{1}{L\omega} \Rightarrow I_m = \frac{U_m}{L\omega}$

$\forall U_m$ $\omega \to +\infty$ $I_m \to 0$ bobine équivalent à 1 interrupteur ouvert.

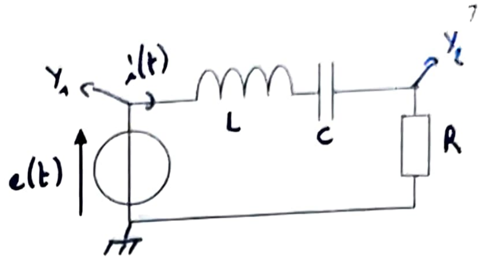
Condensateur parfait $|\underline{Z_C}| = \frac{U_m}{I_m} = \frac{1}{C\omega}$ $\Rightarrow U_m = \frac{I_m}{C\omega}$

$\forall I_m$ $\omega \to +\infty$ $U_m \to 0$ Condensateur équivalent à un fil.

u → 0   f(u) → +∞

III Le circuit RLC série. Etude de l'intensité

1.) Notation complexe



$e(t) = E_m \cos(\omega t)$ (e pris ≡ ref des phases)

On cherche $i(t) = I_m \cos(\omega t + \varphi_i)$

$\sim \underline{e} = E_m e^{j\omega t}$

$\underline{i} = \underline{I_m} e^{j\omega t}$ où $\underline{I_m} = I_m e^{j\varphi_i}$ (p5)

eq de maille : $\boxed{\underline{e} = \underline{Z_L}\,\underline{i} + \underline{Z_C}\,\underline{i} + \underline{Z_R}\,\underline{i}}$ ①

$\underline{e} = \underline{Z_{eq}} \times \underline{i}$

où $\underline{Z_{eq}} = \underline{Z_L} + \underline{Z_C} + \underline{Z_R}$ (impédance en série)

$= jL\omega + \frac{1}{jC\omega} + R$

$= R + jL\omega + \frac{j}{j^2 C\omega}$

$= R + jL\omega - \frac{j}{C\omega}$

$$\boxed{\underline{Z_{eq}} = R + j\left(L\omega - \frac{1}{C\omega}\right)}$$

$\underline{e} = \underline{Z_{eq}}\,\underline{i} \Rightarrow \underline{i} = \frac{\underline{e}}{\underline{Z_{eq}}}$

$\Rightarrow \underline{I_m} e^{j\omega t} = \frac{E_m}{\underline{Z_{eq}}} e^{j\omega t}$

$$\Rightarrow \boxed{\underline{I_m} = \frac{E_m}{\underline{Z_{eq}}} = \frac{E_m}{R + j\left(L\omega - \frac{1}{C\omega}\right)}} \quad ②$$

$I_m = |\underline{I_m}| = \left|\frac{E_m}{\underline{Z_{eq}}}\right| = \frac{E_m}{|\underline{Z_{eq}}|}$

$$\boxed{I_m = \frac{E_m}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}} \quad ③$$

$\varphi_i = \arg(\underline{I_m}) = \arg\left(\frac{E_m}{\underline{Z_{eq}}}\right)$

$= \arg(E_m) - \arg(\underline{Z_{eq}})$

$= 0 - \arg(\underline{Z_{eq}})$

$$\boxed{\varphi_i = -\varphi_{\underline{Z_{eq}}}}$$

Rq1 $\underline{z} = x + jy$ $|\underline{z}| = \sqrt{x^2 + y^2}$

$\underline{z} = |\underline{z}| e^{j\varphi} = |\underline{z}|(\cos\varphi + j\sin\varphi)$

$= |\underline{z}|\cos\varphi + j|\underline{z}|\sin\varphi$

$\Rightarrow x = |\underline{z}|\cos\varphi$, $y = |\underline{z}|\sin\varphi$ $\Rightarrow \cos\varphi = \frac{x}{|\underline{z}|}$, $\sin\varphi = \frac{y}{|\underline{z}|}$

$\tan\varphi = \frac{y}{x}$

$\underline{Z_{eq}} = R + j\left(L\omega - \frac{1}{C\omega}\right)$

$\tan\varphi_{\underline{Z_{eq}}} = \frac{L\omega - \frac{1}{C\omega}}{R}$

$\varphi_{\underline{Z_{eq}}} = \text{Arctan}\left(\frac{L\omega - \frac{1}{C\omega}}{R}\right)$

⚠ tan est périodique de période π + vérifier le signe du cos

$$\boxed{\varphi_i = -\varphi_{\underline{Z_{eq}}} = -\text{Arctan}\left(\frac{L\omega - \frac{1}{C\omega}}{R}\right)} \quad ④$$

$$\boxed{\cos\varphi_{\underline{Z_{eq}}} = \frac{R}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} \geq 0}$$

cos $\varphi_{\underline{Z_{eq}}} \in \left[-\frac{\pi}{2} ; \frac{\pi}{2}\right]$

Rq1: Obtention de l'equa diff

① $\underline{e} = \underline{Z_L}\,\underline{i} + \underline{Z_R}\,\underline{i} + \underline{Z_C}\,\underline{i}$

$= jL\omega\,\underline{i} + R\,\underline{i} + \frac{1}{jL\omega}\,\underline{i}$

$= L\frac{d\underline{i}}{dt} + R\,\underline{i} + \frac{1}{C}\int \underline{i}\,dt$

On dérive : $\frac{d\underline{e}}{dt} = L\frac{d^2\underline{i}}{dt^2} + R\frac{d\underline{i}}{dt} + \frac{\underline{i}}{C}$

$\frac{d^2\underline{i}}{dt^2} + \frac{R}{L}\frac{d\underline{i}}{dt} + \frac{1}{LC}\underline{i} = \frac{1}{L}\frac{d\underline{e}}{dt}$

On prend la partie réelle

$$\boxed{\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = \frac{1}{L}\frac{de}{dt}}$$

formes canoniques

ⓐ $\frac{d^2 i}{dt^2} + 2\lambda\frac{di}{dt} + \omega_0^2 i = \frac{1}{L}\frac{de}{dt}$

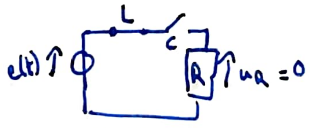
ⓑ $\frac{d^2 i}{dt^2} + \frac{\omega_0}{Q}\frac{di}{dt} + \omega_0^2 i = \frac{1}{L}\frac{de}{dt}$

$\omega_0 = \frac{1}{\sqrt{LC}}$ $\frac{\omega_0}{Q} = \frac{R}{L} \Rightarrow Q = \frac{L\omega_0}{R}$

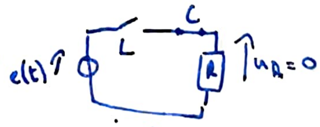
Rq2: Schémas équivalents

à basses fréquences $f \to 0$ $\omega \to 0$ ($\omega = 2\pi f$) (régime continu)

à hautes fréquences $f \to +\infty$ $\omega \to +\infty$



$i = 0$



$i = 0$

2) Étude de l'amplitude de l'intensité

③ $$\boxed{I_m = \frac{E_m}{|\underline{Z}_{eq}|} = \frac{E_m}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}}$$

$|\underline{Z}_{eq}| = \sqrt{f(\omega)}$ où $f(\omega) = R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2$

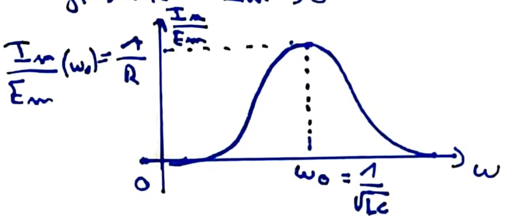
$f'(\omega) = 2\left(L + \frac{1}{C\omega^2}\right)\left(L\omega - \frac{1}{C\omega}\right)$

$f'(\omega) = 0$ pour $L\omega - \frac{1}{C\omega} = 0$

$\Rightarrow \omega^2 = \frac{1}{LC}$

$\Rightarrow \omega = \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$ pulsation propre

- $\omega \to 0$ $f \to 0$ $L\omega \to 0$ $\frac{1}{C\omega} \to +\infty$
  $f(\omega) \to +\infty$ $I_m \to 0$
- $\omega \to +\infty$ $f \to +\infty$ $L\omega \to +\infty$ $\frac{1}{C\omega} \to 0$
  $f(\omega) \to +\infty$ $I_m \to 0$



$f(\omega_0) = R^2 \Rightarrow I_m = \frac{E_m}{\sqrt{R^2}} = \frac{E_m}{R}$

Le max est appelé <u>résonance d'intensité</u>

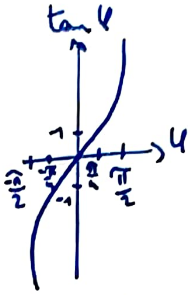
$I_{m\,r} = I_m(\omega_0) = \frac{E_m}{R}$ avec $\boxed{\omega_r = \omega_0}$

<u>pulsation propre de la résonance</u>

3) Étude de la phase

$$\boxed{\varphi_i = -\varphi_{\underline{Z}eq} \text{ où } \varphi_{\underline{Z}eq} = \text{Arctan}\left(\frac{L\omega - \frac{1}{C\omega}}{R}\right)}$$ ④

tan est π-périodique



$\tan\varphi_{\underline{Z}eq} = \frac{L\omega - \frac{1}{C\omega}}{R}$

$\omega \to 0$ $L\omega \to 0$

$\frac{1}{C\omega} \to +\infty$

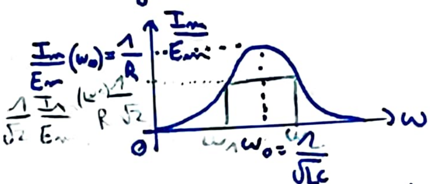
$\tan(\varphi_{\underline{Z}eq}) \to -\infty$ $\varphi_{\underline{Z}eq} = -\frac{\pi}{2}$ $\varphi_i = +\frac{\pi}{2}$

$\omega \to +\infty$ $L\omega \to +\infty$ $\frac{1}{C\omega} \to 0$

$\tan\varphi_{\underline{Z}eq} \to +\infty$ $\varphi_{\underline{Z}eq} \to \frac{\pi}{2}$ $\varphi_i = -\frac{\pi}{2}$

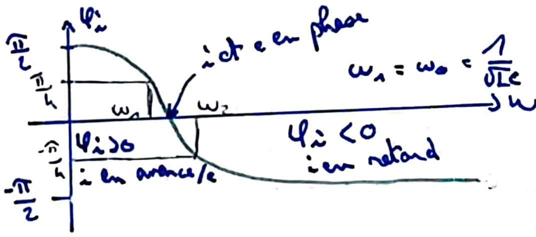
$\omega = 2\pi f$

- $\omega \to 0$ ($f \xrightarrow{BF} 0$) $L\omega \to 0$ $\frac{1}{C\omega} \to +\infty$
  $f(\omega) \to +\infty$ $I_m \to 0$
- $\omega \to +\infty$ ($f \xrightarrow{HF} +\infty$) $L\omega \to +\infty$ $\frac{1}{C\omega} \to 0$
  $f(\omega) \to +\infty$ $I_m \to 0$



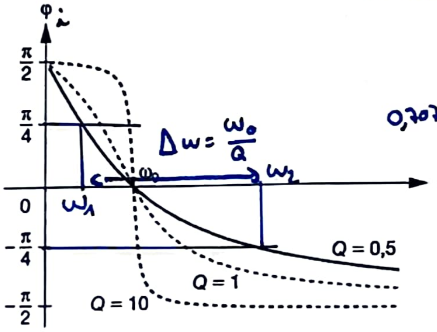
- $\tan\varphi_{\underline{Z}eq} = 0$ pour $L\omega - \frac{1}{C\omega} = 0 \Rightarrow \omega = \omega_0 = \frac{1}{\sqrt{LC}}$

$\Rightarrow \varphi_{\underline{Z}eq} = 0 \Rightarrow \varphi_i = 0$



4) Étude de la bande passante $[\omega_1, \omega_2]$

$\boxed{\text{Intervalle de pulsation pour lequel } I_m \geq \frac{I_{m\,r}}{\sqrt{2}} \text{ où } I_{m\,r} \text{ est l'amplitude de l'intensité de résonance}}$



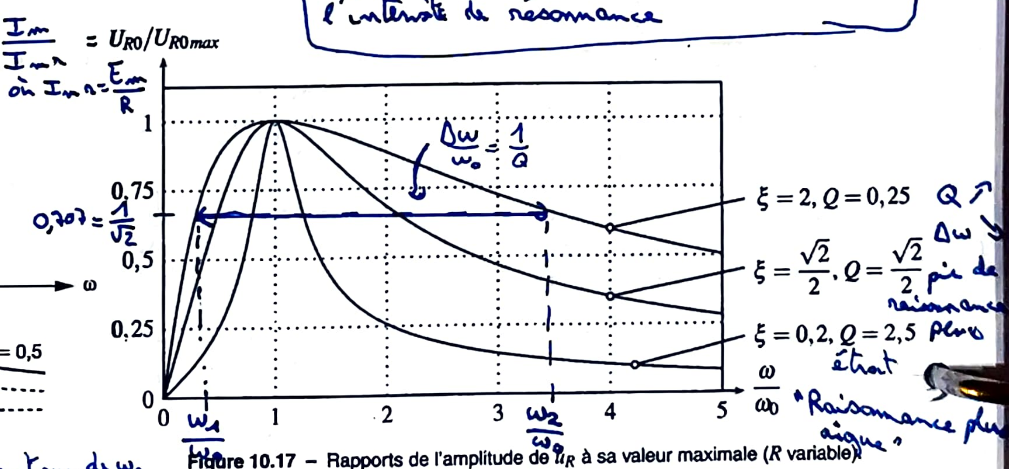


Figure 10.17 – Rapports de l'amplitude de u_R à sa valeur maximale (R variable)

$Q \nearrow \Delta\omega \searrow$

- "Rotation de phase + rapide" autour de ω_0

$$I_{m1} = I_m(\omega_1 = \omega_0) = \frac{E_m}{R}$$

cf graphe 10.17.

③ $$I_m = \frac{E_m}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

Limites de la bande passante $[\omega_1, \omega_2]$
= <u>pulsation de coupure</u>.

$$I_m = \frac{I_{m1}}{\sqrt{2}} \Rightarrow \frac{E_m}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} = \frac{E_m}{R\sqrt{2}}$$

$$\Rightarrow \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2} = R\sqrt{2}$$

$$\Rightarrow R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2 = R^2 \times 2$$

$$\Rightarrow R^2 = \left(L\omega - \frac{1}{C\omega}\right)^2$$

$$\Rightarrow L\omega - \frac{1}{C\omega} = \pm R \quad ⑤ \Delta$$

cf 3) $$\tan \varphi_{Z_{eq}} = \frac{L\omega - \frac{1}{C\omega}}{R} = \pm\frac{R}{R}$$

$$\Rightarrow \tan \varphi_{Z_{eq}} = \pm 1 \Rightarrow \tan \varphi_{Z_{eq}} = \pm\frac{\pi}{4}$$

$$\varphi_i = -\varphi_{Z_{eq}} = \pm\frac{\pi}{4}$$

⑤ $\times C\omega$ $\Rightarrow LC\omega^2 - 1 = \pm RC\omega$

$\Rightarrow LC\omega^2 \pm RC\omega - 1 = 0$

Discriminant $\Delta = R^2C^2 + 4LC > 0$

$$\omega_{1} = \frac{\pm RC \pm \sqrt{\Delta}}{2LC} = \frac{\pm RC \pm \sqrt{R^2C^2 + 4LC}}{2LC}$$

$\sqrt{\Delta} > RC$ $$\omega_1 = \frac{-RC + \sqrt{R^2C^2 + 4LC}}{2LC}$$

$$\omega_2 = \frac{RC + \sqrt{R^2C^2 + 4LC}}{2LC}$$

ω_1 et ω_2 sont > 0

$$\Delta\omega = \omega_2 - \omega_1 = \frac{2RC}{2LC} = \frac{R}{L}$$

(Rq : $\Delta\omega = \frac{R}{L} = \frac{\omega_0}{Q}$
$\Rightarrow Q, \frac{L\omega_0}{R}$

$\boxed{\Delta\omega = \frac{R}{L}}$ Largeur de la bande passante

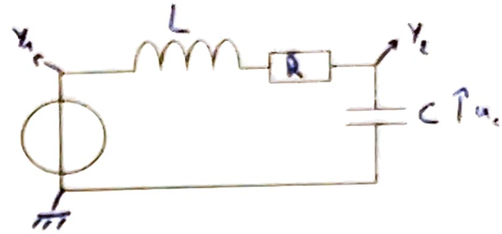
$$\frac{\Delta\omega}{\omega_0} = \frac{R}{L}\sqrt{LC} \quad \text{ou } \omega_0 = \frac{1}{\sqrt{LC}} \qquad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\frac{\Delta\omega}{\omega_0} = R\sqrt{\frac{C}{L}} \Rightarrow \boxed{\frac{\Delta\omega}{\omega_0} = \frac{1}{Q}}$$ ***

sans dim

IV Le circuit RLC série. Etude de la tension aux bornes du condensateur

1.) Notation complexe



$e(t) = E_m \cos(\omega t \ \ ) \rightsquigarrow \underline{e} = E_m e^{j\omega t}$

$u_c(t) = U_{cm} \cos(\omega t + \varphi_u) \rightsquigarrow \underline{u_c} = \underline{U_{cm}}\, e^{\omega t}$ ou $\underline{U_{cm}} = U_{cm} e^{j\varphi_u}$

$$\left.\begin{aligned} \underline{e} &= \left(R + jL\omega + \frac{1}{jC\omega}\right)\underline{i} \\ \underline{u_c} &= \frac{1}{jC\omega}\underline{i} \end{aligned}\right\} \boxed{\frac{\underline{u_c}}{\underline{e}} = \frac{\frac{1}{jC\omega}}{R + jL\omega + \frac{1}{jC\omega}}} \quad (1)$$

Pont diviseur de tension (m̂ courant qui traverse les dipôles)

$$(1) \times \frac{jC\omega}{jC\omega} \quad \frac{\underline{u_c}}{\underline{e}} = \frac{1}{RjC\omega + j^2 LC\omega^2 + 1}$$

$$\boxed{\frac{\underline{u_c}}{\underline{e}} = \frac{1}{1 - LC\omega^2 + RjC\omega}} \quad (2)$$

Rq 1 : Obtention de l'équa diff

$$(1) \quad \underline{u_c} \times \left(R + jL\omega + \frac{1}{jC\omega}\right) = \underline{e} \times \frac{1}{jC\omega}$$

$$R\underline{u_c} + L\frac{d\underline{u_c}}{dt} + \frac{1}{C}\int \underline{u_c}\, dt = \frac{1}{C}\int \underline{e}\, dt$$

on dérive.

$$L\frac{d^2\underline{u_c}}{dt^2} + R\frac{d\underline{u_c}}{dt} + \frac{1}{C}\underline{u_c} = \frac{1}{C}\underline{e}$$

$$\Rightarrow \frac{d^2\underline{u_c}}{dt^2} + \frac{R}{L}\frac{d\underline{u_c}}{dt} + \frac{1}{CL}\underline{u_c} = \frac{1}{CL}\underline{e}$$

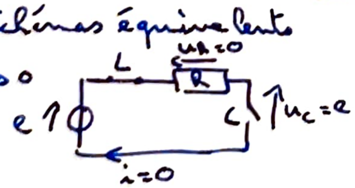
$$\mathrm{Re}(\ ) \Rightarrow \boxed{\frac{d^2u_c}{dt^2} + \frac{R}{L}\frac{du_c}{dt} + \frac{1}{CL}u_c = \frac{1}{CL}e}$$

forme canonique : $\frac{d^2u_c}{dt^2} + \frac{\omega_0}{Q}\frac{du_c}{dt} + \omega_0^2 u_c = \omega_0 e$

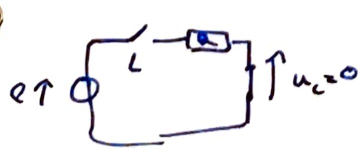
$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \quad \frac{\omega_0}{Q} = \frac{R}{L} \Rightarrow \boxed{Q = \frac{L\omega_0}{R}}$$

Rq 2 : Schémas équivalents

BF $\omega \to 0$



HF : $\omega \to +\infty$



Mise sous forme canonique : $\omega_0^2 = \frac{1}{LC} \Rightarrow LC = \frac{1}{\omega_0^2}$

$$(2) \quad \frac{\underline{u_c}}{\underline{e}} = \frac{1}{1 - \frac{\omega^2}{\omega_0^2} + jRC\omega_0 \times \frac{\omega}{\omega_0}}$$

$\boxed{x = \frac{\omega}{\omega_0}}$ pulsation réduite (sans dim)

$$Q = \frac{L\omega_0}{R} \times \frac{\omega_0}{\omega_0} = \frac{L}{RLC\omega_0} = \frac{1}{RC\omega_0}$$

$$\Rightarrow \boxed{\frac{\underline{u_c}}{\underline{e}} = \frac{1}{1 - x^2 + j\frac{x}{Q}}} \quad (3)$$

$$\frac{\underline{u_c}}{\underline{e}} = \frac{\underline{U_{cm}}\, e^{j\omega t}}{E_m e^{j\omega t}} = \boxed{\frac{\underline{U_{cm}}}{E_m} = \frac{1}{1 - x^2 + j\frac{x}{Q}}}$$

$$(4) \quad \boxed{\frac{U_{cm}}{E_m} = \left|\frac{\underline{U_{cm}}}{E_m}\right| = \frac{1}{\sqrt{(1-x^2)^2 + \frac{x^2}{Q^2}}}}$$

$$\varphi_u = \arg(\underline{U_{cm}}) = \arg\left(\frac{E_m}{1 - x^2 + j\frac{x}{Q}}\right)$$

$$\varphi_u = 0 - \arg\left(1 - x^2 + j\frac{x}{Q}\right)$$

$\boxed{\underline{Den} = 1 - x^2 + j\frac{x}{Q}}$ $\boxed{\varphi_u = \varphi_{Den}}$ où $\varphi_{Den} = \arg(\underline{Den})$

2) Étude de l'amplitude U_{cm}

$$(4) \quad \frac{U_{cm}}{E_m} = \frac{1}{\sqrt{(1-x^2)^2 + \frac{x^2}{Q^2}}} \quad \text{où } x = \frac{\omega}{\omega_0} \quad \frac{U_{cm}}{E_m} = \frac{1}{|\underline{Den}|}$$

$\omega \to 0$ $\quad x \to 0$ $\quad |\underline{Den}| \to 1$ $\quad \frac{U_{cm}}{E_m} \to 1$

$\omega \to +\infty$ $\quad x \to +\infty$ $\quad |\underline{Den}| \to +\infty$ $\quad \frac{U_{cm}}{E_m} \to 0$

$\omega = \omega_0$ $\quad x = 1$

$$|\underline{Den}| = \sqrt{(1-1^2)^2 + \frac{1}{Q^2}} = \frac{1}{Q} \quad \boxed{\frac{U_{cm}}{E_m} = Q}$$

Rq : Étude du maximum $\frac{U_{cm}}{E_m} = \frac{1}{\sqrt{g(x)}}$

où $g(x) = (1-x^2)^2 + \frac{x^2}{Q^2}$ $\quad X = x^2$

$\Rightarrow g(X) = (1-X)^2 + \frac{X}{Q^2}$

$g'(X) = -2(1-X) + \frac{1}{Q^2}$

$g'(X) = 0 \Rightarrow 2(1-X) = \frac{1}{Q^2}$

$\Rightarrow 1 - X = \frac{1}{2Q^2}$

$\Rightarrow \boxed{X = 1 - \frac{1}{2Q^2}}$

$\Rightarrow X = x^2 = 1 - \frac{1}{2Q^2}$

x est réel si $1 - \frac{1}{2Q^2} \geq 0$

$\Rightarrow 1 \geq \frac{1}{2Q^2}$

$\Rightarrow Q^2 \geq \frac{1}{2}$

$\Rightarrow Q \geq \frac{1}{\sqrt{2}}$

$x = \sqrt{1 - \frac{1}{2Q^2}}$

$x = \frac{\omega}{\omega_0} = \sqrt{1 - \frac{1}{2Q^2}} \Rightarrow \boxed{\omega_r = \omega_0 \sqrt{1 - \frac{1}{2Q^2}} < \omega_0}$

pulsat° de raisonance

1 pour $X_r = 1 - \frac{1}{2Q^2}$, $g(x) = \left(1 - 1 + \frac{1}{2Q^2}\right)^2 + \frac{1}{Q^2}\left(1 - \frac{1}{2Q^2}\right)$

$g(x) = \frac{1}{4Q^4} + \frac{1}{Q^2} - \frac{1}{2Q^4} = \frac{4Q^2 - 1}{4Q^4}$

$\frac{U_{cm}}{E_m}(x_r) = \frac{1}{\sqrt{g(x)}} = \sqrt{\frac{4Q^4}{4Q^2-1}} = \frac{2Q^2}{\sqrt{4Q^2-1}}$

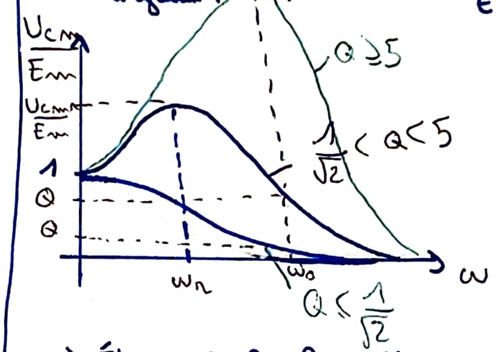
U_{cmr} amplitude de la tension aux bornes du condensateur à la raisonance.

$\boxed{\frac{U_{cmr}}{E_m} = \frac{2Q^2}{\sqrt{4Q^2-1}} > Q}$ pour $\boxed{\omega_r = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}}$ $\quad \omega_r < \omega_0$

pour $Q \geq \frac{1}{\sqrt{2}}$

Rq : $Q = \frac{1}{2}$ $\quad \omega_r = 0$

Q — Q grand ($Q \geq 5$) $\omega_r \simeq \omega_0$ $\quad \frac{U_{cmr}}{E_m} \simeq Q$



3) Étude de la phase φ_u

$\varphi_u = -\varphi_{Den}$ où $\underline{Den} = 1 - x^2 + j\frac{x}{Q}$ et $\varphi_{Den} = \arg(\underline{Den})$

$\cos\varphi_{Den} = \frac{1-x^2}{|\underline{Den}|}$ $\quad \sin\varphi_{Den} = \frac{\frac{x}{Q}}{|\underline{Den}|}$

$\boxed{\tan\varphi_{Den} = \frac{\sin\varphi_{Den}}{\cos\varphi_{Den}} = \frac{x}{Q(1-x^2)}}$

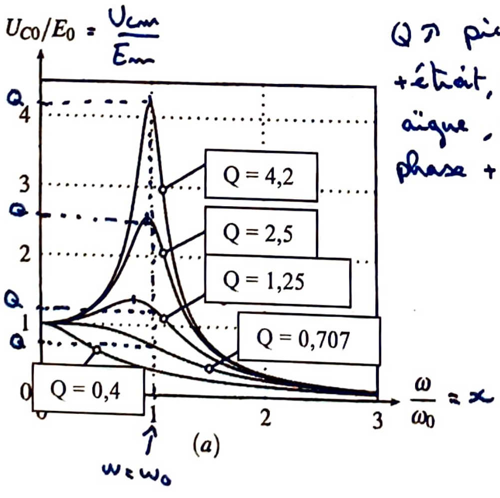
BF $x \to 0$ $\quad \omega \to 0$ $\quad \cos\varphi_{Den} > 0$

$\varphi_{Den} \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$

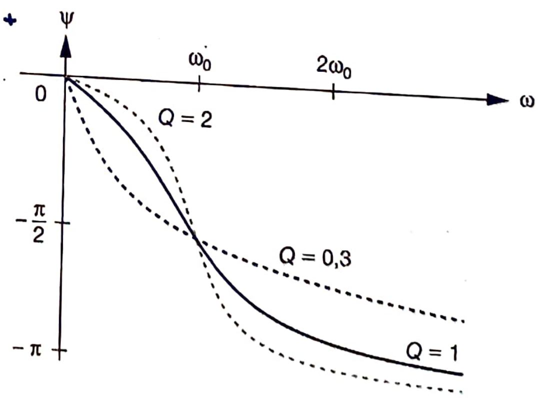
$\tan\varphi_{Den} \to 0$ $\quad \varphi_{Den} \to 0$

$\varphi_u \to 0$

Figure 10.19 – Étude de la résonance aux bornes du condensateur



Q ↗ pic de raisonance + étroit, raisonance + aigüe, rotat° de phase + rapide



HF $x \to +\infty$ $\omega \to +\infty$ $\cos \varphi_{Den} < 0$

$\varphi_{Den} \in \left[\frac{\pi}{2} ; \frac{3\pi}{2}\right]$ ⚠

$\tan \varphi_{Den} \to 0$ $\varphi_{Den} \to 0 \, (\pi)$

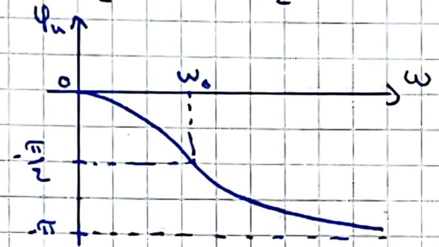
$\varphi_{Den} \to \pi$ $\varphi_u \to -\pi$

$\omega = \omega_0$ $x = 1$ $|\underline{Den}| = \sqrt{(1-x^2)^2 + \frac{x^2}{Q^2}}$

$x = 1$ $|\underline{Den}| = \sqrt{\frac{1}{Q^2}} = \frac{1}{Q}$

$\cos \varphi_{Den} = 0$ $\sin \varphi_{Den} = \frac{1}{Q} \times Q = 1$

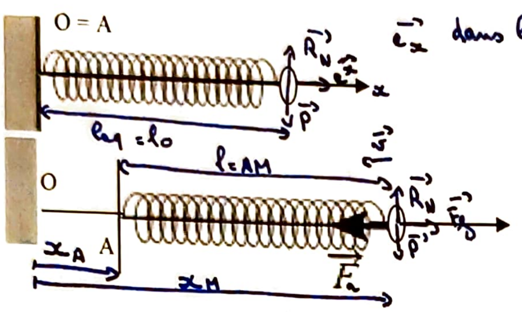
$\varphi_{Den} = \frac{\pi}{2}$ $\varphi_u = \frac{-\pi}{2}$



V L'oscillateur harmonique amorti

1) Equation du mouvement

https://phyanim.sciences.univ-nantes.fr/Meca/Oscillateurs/ressort_rsf.php



$\vec{e_x}$ dans le sens d'étirement du ressort

Syst [anneau M(m)] Ref terrestre galiléen

forces : Poids $\vec{P} = m\vec{g}$, $\vec{R_N}$ réact° normale, Force de rappel du ressort $\vec{F_r} = -k(l-l_0)\vec{e_x}$, Force de frottement fluides $\vec{F_f} = -\alpha\vec{v}$

À l'équilibre $\sum \vec{F} = \vec{0} \Rightarrow \vec{P} + \vec{R_N} + \vec{F_r} = 0$

sur (Ox) $-k(l_{eq} - l_0) = 0 \Rightarrow l_{eq} = l_0$

en mvt : Un dispositif entre O et A (O pt fixe sur le mur, A une extrémité du ressort) permet un déplacement sinusoïdal de A.

$\boxed{x_A = A\cos(\omega t)}$ on cherche $x_M(t)$

2^e loi de Newton : $\sum \vec{F} = m\vec{a}$

$\vec{OM} = x_M\vec{e_x}$ $\quad \vec{v} = \dot{x}_M\vec{e_x}$ $\quad \vec{a} = \ddot{x}_M\vec{e_x}$

$\vec{P} + \vec{R_N} + \vec{F_r} + \vec{F_f} = m\vec{a}$

sur (Ox) : $m\ddot{x}_M = -k(l-l_0) - \alpha\dot{x}_M$

$= -k[(x_M - x_A) - l_0] - \alpha\dot{x}_M$

$\Rightarrow m\ddot{x}_M = kx_A - kx_M + kl_0 - \alpha\dot{x}_M$

$m\ddot{x}_M + k(x_M - l_0) + \alpha\dot{x}_M = kx_A$

$x_1(t) = x_M(t) - l_0 \Rightarrow \dot{x}_1 = \dot{x}_M$ et $\ddot{x}_1 = \ddot{x}_M$

$\Rightarrow \boxed{\ddot{x}_1 + \frac{\alpha}{m}\dot{x}_1 + \frac{k}{m}x_1 = \frac{k}{m}x_A}$ forme canonique

① $\ddot{x}_1 + \frac{\omega_0}{Q}\dot{x}_1 + \omega_0^2 x = \omega_0^2 x_A$

où $\frac{\alpha}{m} = \frac{\omega_0}{Q}$ et $\frac{k}{m} = \omega_0^2 \Rightarrow Q = \frac{m\omega_0}{\alpha}$ et $\omega_0 = \sqrt{\frac{k}{m}}$

Equa diff linéaire : la solut° est du m̂ type que le second membre

$\boxed{x_1(t) = X\cos(\omega t + \varphi)}$

2) Réponse en élongat° : étude de $x_1(t)$

$x_A = A\cos(\omega t) \sim \underline{x_A} = Ae^{j\omega t}$

$x_1 = X\cos(\omega t + \varphi) \sim \underline{x_1} = \underline{X}e^{j\omega t}$ où $\underline{X} = Xe^{j\varphi}$

① $\underline{\ddot{x}_1} + \frac{\omega_0}{Q}\underline{\dot{x}_1} + \omega_0^2\underline{x_1} = \omega_0^2\underline{x_A}$

$\Rightarrow (j\omega)^2\underline{x_1} + \frac{\omega_0}{Q}j\omega\underline{x_1} + \omega_0^2\underline{x_1} = \omega_0^2\underline{x_A}$

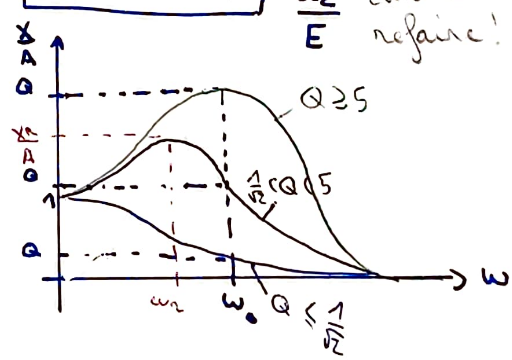
$\Rightarrow \underline{x_1}\left[-\omega^2 + \omega_0^2 + \frac{j\omega\omega_0}{Q}\right] = \omega_0^2\underline{x_A}$

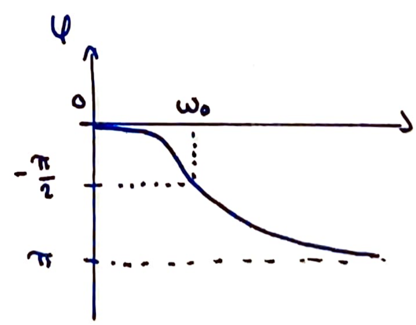
$\Rightarrow \underline{X} = \frac{\omega_0^2}{\omega_0^2 - \omega + \frac{j\omega\omega_0}{Q}}A$ en simplifiant par $e^{j\omega t}$

On divise par ω_0^2 $\quad \underline{X} = \frac{A}{1 - \frac{\omega^2}{\omega_0^2} + j\frac{\omega}{Q\omega_0}}$

pulsat° réduite : $u = \frac{\omega}{\omega_0}$ $\quad \underline{X} = \frac{A}{1 - u^2 + j\frac{u}{Q}}$

$\Rightarrow \boxed{\frac{\underline{X}}{A} = \frac{1}{1 - u^2 + j\frac{u}{Q}}}$ m̂ expression que $\frac{u_c}{E}$ calculs à refaire !





3) Réponse en vitesse

a) Notat° complexe

vitesse $v = \dot{x}_1$; $\underline{v} = \dot{\underline{x}}_1 = j\omega \underline{x}_1$ $\underline{v} = \underline{V} e^{j\omega t}$ où $\underline{V} = V e^{j\varphi_v}$

Donc $\underline{V} e^{j\omega t} = j\omega \underline{X} e^{j\omega t}$ -> $\boxed{\underline{V} = j\omega \underline{X}}$

① => $\underline{V} = \dfrac{j\omega A}{1-u^2+j\frac{u}{Q}}$ $u = \dfrac{\omega}{\omega_0}$ => $\omega = u\omega_0$

=> $\boxed{\underline{V} = \dfrac{j\omega_0 A u}{1-u^2+j\frac{u}{Q}}}$ On divise par ju

$$\underline{V} = \frac{\omega_0 A}{\frac{1}{ju} - \frac{u^2}{ju} + \frac{ju}{juQ}} = \frac{\omega_0 A}{\frac{1}{ju} + ju + \frac{1}{Q}}$$

Donc $\underline{V} = \dfrac{\omega_0 A}{-\frac{j}{u} + ju + \frac{1}{Q}} = \dfrac{\omega_0 A}{\frac{1}{Q} + j(u-\frac{1}{u})} = \dfrac{Q\omega_0 A}{1 + Qj(u+\frac{1}{u})}$

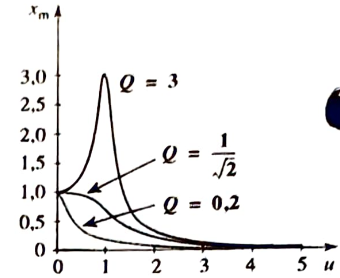
③ $\boxed{\underline{V} = \dfrac{\omega_0 A Q}{1 + jQ(u-\frac{1}{u})}}$ étude possible à partir du ②

forme canonique du ③

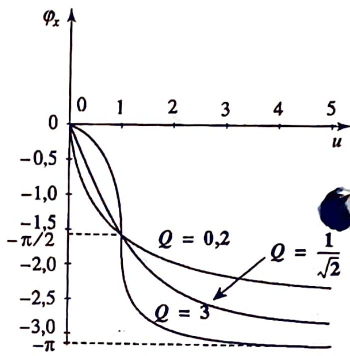
b) étude de l'amplitude de la vitesse

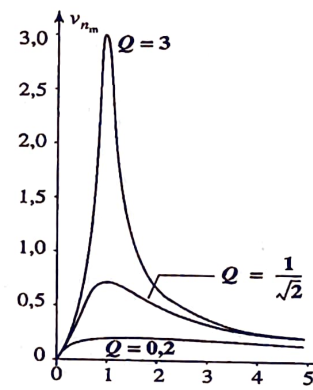
③ $\left|\dfrac{\underline{V}}{A}\right| = \dfrac{V}{A} = \dfrac{\omega_0 Q}{\sqrt{1+Q^2(u-\frac{1}{u})^2}} = \dfrac{\omega_0 Q}{\sqrt{f(u)}}$

$f(u) = 1 + Q^2(u-\frac{1}{u})^2$



Doc. 8. *Variation de l'amplitude normalisée $x_{n_m} = \dfrac{x_m}{x_{A_m}}$ de la réponse en élongation en fonction de $u = \dfrac{\omega}{\omega_0}$ pulsation normalisée de l'excitation pour différents amortissements.*





Doc. 12. *Variation de l'amplitude : $v_{n_m} = \dfrac{v_m}{\omega_0 x_{A_m}}$ de la réponse en vitesse en fonction de la pulsation normalisée $u = \dfrac{\omega}{\omega_0}$ de l'excitation*