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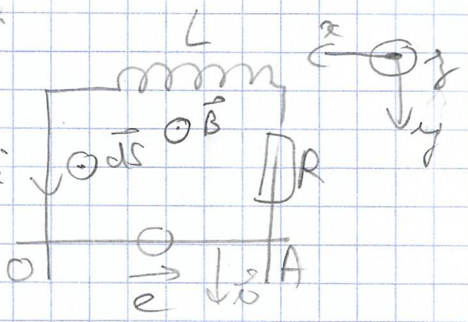
TD MA3 Circuit mobile dans $\vec{B}$

Exo 1 - Tige qui chute

1) Loi de Faraday



La tige coupe les lignes de champ de $\vec{B}$

Donc $e = -\frac{d\varphi}{dt} = -\frac{d}{dt}(Bay) = -Ba\frac{dy}{dt} = -Bav$

éq de maille $\quad Ri + L\frac{di}{dt} = e$

$$\Rightarrow \boxed{L\frac{di}{dt} + Ri = -Bav} \quad (EE)$$

Résultante des forces de Laplace $\quad \vec{f}_L = i\,\vec{OA}\wedge\vec{B} = iaB\,\vec{u_y}$

Équation mécanique $\quad \boxed{m\frac{dv}{dt} = mg + iaB} \quad (EM)$

On dérive (EE) : $\quad L\frac{d^2i}{dt^2} + R\frac{di}{dt} = -Ba\frac{dv}{dt}$

(EM)

$$\Rightarrow L\frac{d^2i}{dt^2} + R\frac{di}{dt} = -Ba\left(g + \frac{iaB}{m}\right)$$

$$\Rightarrow \boxed{\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{B^2a^2}{mL}i = -\frac{Bag}{L}}$$

2) $R = 0$

$\frac{d^2i}{dt^2} + \frac{B^2a^2}{mL}i = -\frac{Bag}{L} \qquad \omega^2 = \frac{B^2a^2}{mL} \qquad \omega = \frac{Ba}{\sqrt{mL}}$

$$\left.\begin{array}{l} i_\ell(t) = A_1\cos(\omega t) + B_1\sin(\omega t) \\ i_p = -\frac{Bag}{L}\times\frac{mL}{B^2a^2} \qquad i_p = \frac{-mg}{aB} \end{array}\right\} \; i(t) = A_1\cos(\omega t) + B_1\sin(\omega t) - \frac{mg}{aB}$$

Continuité du courant dans la bobine donc $i(0^+) = 0$

$\Rightarrow A_1 - \frac{mg}{aB} = 0 \qquad A_1 = \frac{mg}{aB}$

(EE) à $t = 0^+$ $\quad L\frac{di}{dt}(0) + R\,i(0) = -Ba\,v(0)$

Or $v(0) = 0 \quad i(0) = 0 \Rightarrow \frac{di}{dt}(0) = 0.$

$\frac{di}{dt} = -A_1\omega\sin(\omega t) + B_1\omega\cos(\omega t) \qquad \frac{di}{dt}(0) = B_1\omega = 0 \Rightarrow B_1 = 0.$

d'où $\boxed{i(t) = \frac{mg}{aB}\left[\cos(\omega t) - 1\right]} \qquad i < 0.$

$$\left(\begin{array}{l} \text{dans (EM)} \quad m\frac{dv}{dt} = mg + mg\left(\cos(\omega t) - 1\right] = mg\cos(\omega t) \\ \Rightarrow \frac{dv}{dt} = g\cos(\omega t) \qquad v = \frac{g}{\omega}\sin(\omega t) + \text{cste} \end{array} \right) \text{ ou (EE).}$$

$v(0) = 0 \Rightarrow \boxed{v(t) = \frac{g}{\omega} \sin(\omega t)}$

$v(t) = \frac{g\sqrt{mL}}{Ba} \sin(\omega t)$

$y(t) = -\frac{g\sqrt{mL}}{Ba} \frac{1}{\omega} \cos(\omega t) + cte \Rightarrow$ oscillations de la barre

à $t = 0$ $y = y_0$

$\boxed{y(t) = \frac{g\sqrt{mL}}{Ba\omega}\left(1 - \cos(\omega t)\right) + y_0}$

3) Si R est grand $\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{B^2 a^2}{mL} i = -\frac{Bag}{L}$ $\quad i_f = -\frac{mg}{aB}$

$\frac{d^2 i}{dt^2} + 2\lambda \frac{di}{dt} + \omega_0^2 i = 0$ $\quad r^2 + 2\lambda r + \omega_0^2 = 0$

$\Delta = 4(\lambda^2 - \omega_0^2)$ Si $\lambda \gg \omega_0$ $\quad \Delta > 0$ Régime apériodique

$\Delta = \frac{R^2}{L^2} - \frac{4B^2a^2}{mL} > 0$ $\quad R > 2aB\sqrt{\frac{L}{m}}$

$i_\ell(t) = A_2 e^{+r_1 t} + B_2 e^{r_2 t} \longrightarrow 0$ rapidement.

$i \to i_f = -\frac{mg}{aB}$ On tend rapidement vers le régime permanent $\frac{di}{dt} = 0$ et $\frac{dv}{dt} = 0$

$\Rightarrow$ (EE) $R i_\infty = -Ba v_\infty$ $\quad v_\infty = -\frac{R i_\infty}{Ba}$

(EM) $0 = mg + i_\infty aB \Rightarrow \boxed{i_\infty = -\frac{mg}{aB}} \rightarrow \boxed{v_\infty = \frac{Rmg}{a^2B^2}}$

La vitesse de chute est limitée, on a freinage par induction