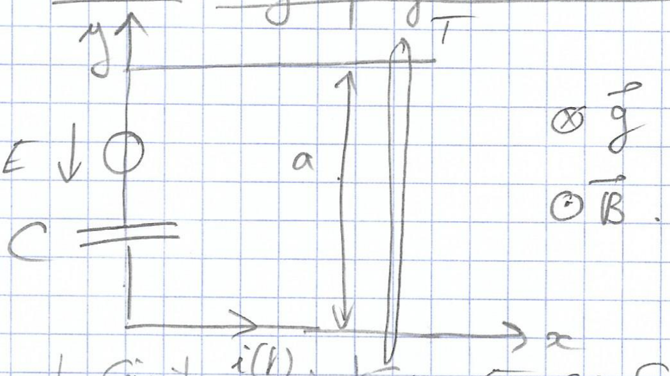
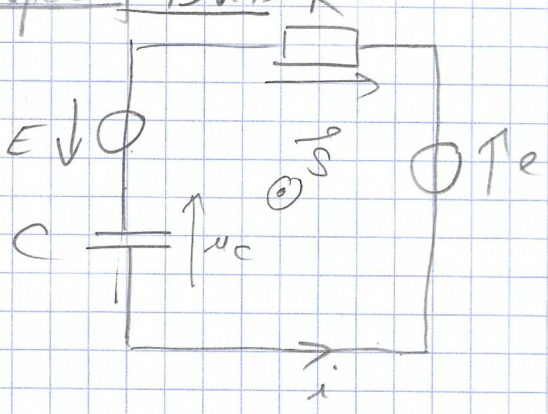
Exo 2 Tige qui glisse sur un circuit capacitif TD MA3





1) Circuit orienté par E en CVG

$\vec{S} = S\,\vec{e_z}$ d'après la règle de la main droite

$\phi = \vec{B}\cdot\vec{S} = Bax$ $\qquad e = -\dfrac{d\phi}{dt} = -Bav$ | fem induite | Loi de Faraday

(EE) $E + e - u_C - Ri = 0$

$\Rightarrow \boxed{E - Bav = Ri + u_C}$ (EE)

Force de Laplace $\vec{F_L} = iaB\,\vec{u_x}$ $\qquad m\vec{a} = \vec{F_L} + \vec{P} + \vec{R_N}$ pour la tige

LFD $\boxed{m\dfrac{dv}{dt} = iaB}$ (EM) $\qquad \boxed{\dfrac{dv}{dt} = \dfrac{iaB}{m}}$

2) On dérive (EE)/t

$-Ba\dfrac{dv}{dt} = R\dfrac{di}{dt} + \dfrac{du_C}{dt}$ $\qquad$ et $i = C\dfrac{du_C}{dt}$

$\Rightarrow -Ba\dfrac{dv}{dt} = R\dfrac{di}{dt} + \dfrac{i}{C} \overset{EM}{\Rightarrow} \dfrac{di}{dt} + \dfrac{i}{RC} = -\dfrac{Ba}{R}\left(\dfrac{iaB}{m}\right)$

$\dfrac{di}{dt} + \left[\dfrac{1}{RC} + \dfrac{B^2a^2}{mR}\right] i = 0 \qquad \dfrac{di}{dt} + \alpha i = 0$

$\alpha = \dfrac{1}{RC} + \dfrac{B^2a^2}{mR} = \dfrac{m + a^2B^2C}{mRC}$ $\qquad \boxed{\tau = \dfrac{1}{\alpha} = \dfrac{mRC}{m + a^2B^2C}}$

$i(t) = A e^{-\alpha t} = A e^{-\frac{t}{\tau}}$

| u_C est continue en $t=0$ $\Rightarrow u_C(0) = 0$
| la tige est initialement immobile $\Rightarrow v(0) = 0$.

$\Rightarrow$ (EE) $\quad E = Ri(0) \Rightarrow \boxed{i(0) = \dfrac{E}{R}} \Rightarrow \boxed{i(t) = \dfrac{E}{R}e^{-\alpha t}}$

3) (EM) $\dfrac{dv}{dt} = \dfrac{aB}{m} \times \dfrac{E}{R} e^{-\frac{t}{\tau}}$

$v = \dfrac{aBE}{mR} \times (-\tau)\, e^{-\frac{t}{\tau}} + cte = -\dfrac{aBE\tau}{mR} e^{-\frac{t}{\tau}} + cte$

à $t=0$ $\quad v(0) = 0 \Rightarrow \boxed{v = \dfrac{aBE\tau}{mR}\left[1 - e^{-\frac{t}{\tau}}\right]}$

4) $\mathcal{E}_{G_{0\to\infty}} = \int_0^\infty \mathcal{P}_G \, dt$ où $\mathcal{P}_G = Ei$ et $i = C\frac{du_c}{dt}$

$= \int_0^\infty E \frac{E}{R} e^{-\frac{t}{\tau}} dt = \frac{E^2}{R}\left[-\tau e^{-\frac{t}{\tau}}\right]_0^\infty = \frac{E^2\tau}{R}$

$= E \times C [u_c]_0^\infty$

5) $u_c(0) = 0$

$i(t) = C\frac{du_c}{dt} = \frac{E}{R}\exp\left(-\frac{t}{\tau}\right) \Rightarrow \boxed{u_c = \frac{E\tau}{RC}\left(1 - \exp\left(-\frac{t}{\tau}\right)\right)}$

6) $\mathcal{E}_{C_{0\to\infty}} = \int_0^\infty u_c i \, dt = C\int_0 u_c \frac{du_c}{dt} dt = \left[\frac{1}{2}Cu_c^2\right]_0^\infty$

$\mathcal{E}_{C_{0\to\infty}} = \frac{1}{2}C\left(u_{c\infty}^2 - u_{c0}^2\right) = \frac{1}{2}C\frac{E^2\tau^2}{R^2C^2}$ $\boxed{\mathcal{E}_{C_{0\to\infty}} = \frac{E^2\tau^2}{2R^2C}}$

7) $\mathcal{E}_{J_{0\to\infty}} = \int_0 Ri^2 dt = \int_0^\infty R \times \frac{E^2}{R^2} e^{-\frac{2t}{\tau}} dt$

$\mathcal{E}_{J_{0\to\infty}} = \frac{E^2}{R}\left[-\frac{\tau}{2}e^{-\frac{2t}{\tau}}\right]_0^\infty = -\frac{E^2\tau}{2R}[0-1]$ $\boxed{\mathcal{E}_J = \frac{E^2\tau}{2R}}$

8) $\mathcal{P}_L = \vec{f}_L \cdot \vec{v} = iaBv$ (EΠ) $m\frac{dv}{dt} = iaB$

(EΠ) $m\frac{dv}{dt}v = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = \mathcal{P}_L$ $\frac{dE_c}{dt} = \mathcal{P}_L$.

$W(\vec{f}_L) = \int_0^\infty \mathcal{P}_L dt = \int_0^\infty \frac{d}{dt}\left(\frac{1}{2}mv^2\right)dt = \left[\frac{1}{2}mv^2_\infty - \frac{1}{2}mv_0^2\right]$

$= \frac{1}{2}m\frac{a^2B^2E^2\tau^2}{m^2R^2}$ $\boxed{W(\vec{f}_L) = \frac{a^2B^2E^2\tau^2}{2mR^2}}$

C'est en fait le théorème de l'Ec

9) $\mathcal{E}_{G_{0\to\infty}} = \mathcal{E}_{C_{0\to\infty}} + \mathcal{E}_{J_{0\to\infty}} + W_{0\to\infty}$ à vérifier. | faire (EE) x i

L'énergie fournie par le géné sert à charger le condensateur à mettre en mouvement la tige et à compenser les pertes Joule.

Vérifier que $\mathcal{E}_{gén_{0\to\infty}} = \mathcal{E}_{C_{0\to\infty}} + \mathcal{E}_{J_{0\to\infty}} + W_{0\to\infty}(\vec{f}_L)$.

$\frac{E^2\tau}{R} = \frac{E^2\tau^2}{2R^2C} + \frac{E^2\tau}{2R} + \frac{a^2B^2E^2\tau^2}{2mR^2}$

$\mathcal{E}_C + W(\vec{f}_L) = \left(\frac{E^2}{2R^2C} + \frac{a^2B^2E^2}{2mR^2}\right)\tau^2$ où $\tau = \frac{mRC}{m + a^2B^2C}$

$\frac{E^2\tau^2}{2R^2C}\left(1 + \frac{a^2B^2C}{m}\right) = \frac{E^2}{2R^2Cm}\left(m + a^2B^2C\right)\tau^2$

$= \frac{E^2}{2R^2C\cancel{m}} \times \cancel{m}RC \times \tau = \frac{E^2\tau}{2R}$) OK

$\mathcal{E}_J + W(\vec{f}_L) + \mathcal{E}_J = \frac{E^2\tau}{2R} + \frac{E^2\tau}{2R} = \frac{E^2\tau}{R} = \mathcal{E}_G$.

(EE) x i : $Ei = Bavi + Ri^2 + u_c i$ $\boxed{\mathcal{P}_{gén} = \mathcal{P}_C + \mathcal{P}_J + \mathcal{P}_L}$