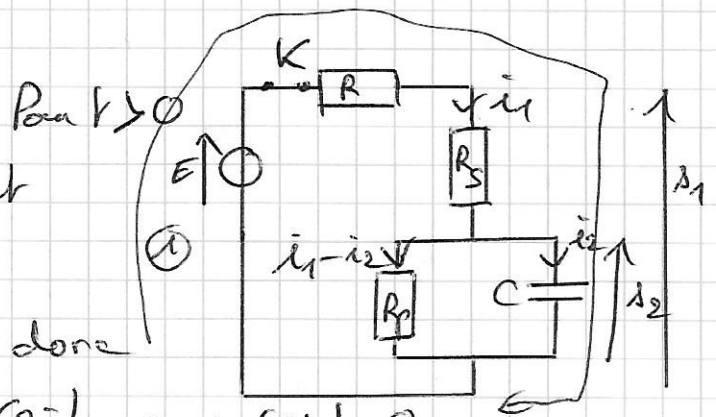


TD SE2 Exercice 4

1) Valeurs à $t=0^+$
Le condensateur est initialement
déchargé donc $s_2(0^-) = 0$

$\mathcal{E}_C = \frac{1}{2} C u_C^2$ est continue, donc
de même : $s_2(0^+)/ = s_2(0^-) \Rightarrow s_2(0^+) = 0$

Pont diviseur de tension $\frac{s_1(0^+)}{E} = \frac{R_S}{R+R_S}$
 $\Rightarrow s_1(0^+) = \frac{R_S}{R+R_S} E$

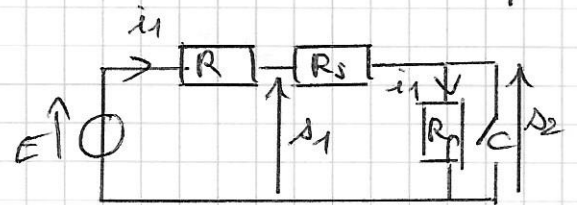


Valeurs à $t \rightarrow \infty$ On est en régime permanent. Le condensateur équivaut
à un interrupteur ouvert.

Pont diviseur de tension :

$$s_1(t_{\infty}) = \frac{R_S + R_P}{R + R_S + R_P} E$$

$$s_2(t_{\infty}) = \frac{R_P}{R + R_S + R_P} E$$



2) Condensateur idéal

Equation de maille $E = R i_1 + s_1$

Où $i_1 = C \frac{ds_1}{dt} \Rightarrow E = RC \frac{ds_1}{dt} + s_1$

$$\Rightarrow \frac{ds_1}{dt} + \frac{1}{RC} s_1 = \frac{E}{RC}$$

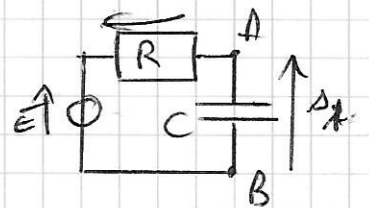
$$s_{1h} = A \exp\left(-\frac{t}{RC}\right)$$

$$s_{1p} = E$$

$$s_1(t) = A \exp\left(-\frac{t}{RC}\right) + E$$

Où $s_1(0^+) = 0 \Rightarrow A = -E$

$$s_1(t) = E \left[1 - \exp\left(-\frac{t}{RC}\right) \right]$$



3) Equation différentielle sur s_2

$$s_2 = R_P(i_1 - i_2)$$

$$\Rightarrow i_1 = \frac{s_2}{R_P} + i_2 \quad (2)$$

Equation de maille (1) : $E = (R + R_S) i_1 + s_2$

$$\Rightarrow E = (R + R_S) \left(\frac{s_2}{R_P} + i_2 \right) + s_2 \quad \text{Où } i_2 = C \frac{ds_2}{dt}$$

$$\Rightarrow E = \frac{R + R_S}{R_P} s_2 + (R + R_S) C \frac{ds_2}{dt} + s_2$$

$$\Rightarrow (R + R_S) C \frac{ds_2}{dt} + \frac{R + R_S + R_P}{R_P} s_2 = E$$

$$\Rightarrow \frac{ds_2}{dt} + \frac{R+R_s+R_p}{CR_p(R+R_s)} s_2 = \frac{E}{(R+R_s)C}$$

$$\frac{ds_2}{dt} + \frac{1}{\tau} s_2 = 0 \quad \text{ou} \quad \tau = \frac{CR_p(R+R_s)}{R+R_s+R_p} \quad s_2(t) = A \exp\left(-\frac{t}{\tau}\right)$$

$$s_{2f} = \frac{R_p E}{R+R_s+R_p}$$

$$s_2 = s_{2f} \left[1 - \exp\left(-\frac{t}{\tau}\right)\right] \quad \text{car } s_2(0^+) = 0$$

$$\textcircled{4} \quad i_2 = C \frac{ds_2}{dt} = \frac{C s_{2f}}{\tau} \exp\left(-\frac{t}{\tau}\right) \quad i_2 = \frac{E}{R+R_s} \exp\left(-\frac{t}{\tau}\right)$$

$$\textcircled{5} \quad \textcircled{2} \quad i_1 = \frac{s_2}{R_p} + i_2 = \frac{E}{R+R_s+R_p} \left[1 - \exp\left(-\frac{t}{\tau}\right)\right] + \frac{E}{R+R_s} \exp\left(-\frac{t}{\tau}\right)$$

$$i_1 = \frac{E}{R+R_s+R_p} \left[1 - \exp\left(-\frac{t}{\tau}\right)\right] + \frac{R+R_s+R_p}{R+R_s} \exp\left(-\frac{t}{\tau}\right)$$

$$i_1 = \frac{E}{R+R_s+R_p} \left[1 + \frac{R_p}{R+R_s} \exp\left(-\frac{t}{\tau}\right)\right]$$

$$\textcircled{6} \quad E = R i_1 + s_1 \Rightarrow s_1 = E - R i_1$$

$$s_1 = E - \frac{R E}{R+R_s+R_p} \left[1 + \frac{R_p}{R+R_s} \exp\left(-\frac{t}{\tau}\right)\right]$$

$$s_1 = E - \frac{R E}{R+R_s+R_p} - \frac{R_p R E}{(R+R_s)(R+R_s+R_p)} \exp\left(-\frac{t}{\tau}\right)$$

$$s_1 = \frac{(R_s+R_p) E}{R+R_s+R_p} - \frac{R_p R E}{(R+R_s)(R+R_s+R_p)} \exp\left(-\frac{t}{\tau}\right)$$

$$s_1(t) = \frac{(R_s+R_p) E}{R+R_s+R_p} \left[1 - \frac{R R_p}{(R_s+R)(R_s+R_p)} \exp\left(-\frac{t}{\tau}\right)\right]$$

On vérifie les valeurs trouvées au [1].

On retrouve avec $R_s = 0$

et $R_p \propto$ les résultats du [2]

