

## Correction du Test n° 3

### Sujet A

1.

2. On pose  $z_1 = -(\sqrt{3} + i)$  et  $z_2 = 1 + i$ .

$$(a) \frac{z_1}{z_2} = \frac{-(\sqrt{3} + i)}{1 + i} \times \frac{1 - i}{1 - i} = \frac{-\sqrt{3} - i + i\sqrt{3} + i^2}{2} = \frac{-1 - \sqrt{3} + i(\sqrt{3} - 1)}{2}$$

$$(b) z_1 = -(\sqrt{3} + i) = 2 \left( -\frac{\sqrt{3}}{2} - \frac{i}{2} \right) = 2e^{-\frac{5i\pi}{6}}$$

$$z_2 = 1 + i = \sqrt{2}e^{\frac{i\pi}{4}}$$

$$Z = \frac{z_1}{z_2} = \frac{2e^{-\frac{5i\pi}{6}}}{\sqrt{2}e^{\frac{i\pi}{4}}} = \sqrt{2}e^{-\frac{5i\pi}{6} - \frac{i\pi}{4}} = \sqrt{2}e^{-\frac{13i\pi}{12}} = \sqrt{2}e^{\frac{11i\pi}{12}}$$

$$(c) \cos\left(\frac{11\pi}{12}\right) = \frac{\operatorname{Re}(Z)}{|Z|} = \frac{-1 - \sqrt{3}}{2\sqrt{2}} \text{ et } \sin\left(\frac{11\pi}{12}\right) = \frac{\operatorname{Im}(Z)}{|Z|} = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

## Correction du Test n° 3

### Sujet B

1.

2. On pose  $z_1 = 1 + i$        $z_2 = \sqrt{3} + i$     et     $Z = z_1^3 z_2$ 

$$(a) z_1^3 = (1 + i)^2(1 + i) = 2i(1 + i) = -2 + 2i$$

$$Z = z_1^3 z_2 = (-2 + 2i)(\sqrt{3} + i) = -2\sqrt{3} - 2i + 2i\sqrt{3} + 2i^2 = -2 - 2\sqrt{3} + i(2\sqrt{3} - 2)$$

$$(b) z_1 = 1 + i = \sqrt{2}e^{\frac{i\pi}{4}} \text{ donc } z_1^3 = \sqrt{2}^3 e^{\frac{3i\pi}{4}} = 2\sqrt{2}e^{\frac{3i\pi}{4}}$$

$$z_2 = \sqrt{3} + i = 2 \left( \frac{\sqrt{3}}{2} + \frac{i}{2} \right) = 2e^{\frac{i\pi}{6}}$$

$$Z = 4\sqrt{2}e^{\frac{3i\pi}{4} + \frac{i\pi}{6}} = 4\sqrt{2}e^{\frac{11i\pi}{12}}$$

$$(c) \cos\left(\frac{11\pi}{12}\right) = \frac{\operatorname{Re}(Z)}{|Z|} = \frac{-1 - \sqrt{3}}{2\sqrt{2}} \text{ et } \sin\left(\frac{11\pi}{12}\right) = \frac{\operatorname{Im}(Z)}{|Z|} = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$