Correction du Test n° 7

Sujet A

1. Résoudre le système (S) : $\left\{\begin{array}{rcrcrcrl} x & - & 2y & + & z & = & 1 & L_1 \\ -2x & + & 3y & - & 3z & = & -1 & L_2 \\ 3x & + & y & + & z & = & -6 & L_3 \end{array}\right. \Leftrightarrow$

$$\left\{\begin{array}{rcrcrcrl} x & - & 2y & + & z & = & 1 & L_1 \\ & - & y & - & z & = & 1 & L_2+2L_1 \\ & & 7y & - & 2z & = & -9 & L_3-L_1 \end{array}\right. \Leftrightarrow \left\{\begin{array}{rcrcrcrl} x & - & 2y & + & z & = & 1 & L_1 \\ & & y & + & z & = & -1 & -L_2 \\ & & & - & 9z & = & -2 & L_3+7L_2 \end{array}\right.$$

$$\Leftrightarrow \left\{\begin{array}{l} x = -\frac{15}{9} = -\frac{5}{3} \\ y = -\frac{11}{9} \\ z = \frac{2}{9} \end{array}\right.$$

2. $f(x) = \dfrac{2+x^2}{x^2} = \dfrac{2}{x^2} + 1$ sur $I =]0, +\infty[$. $\quad F(x) = -\dfrac{2}{x} + x$

3. Calculer $J = \displaystyle\int_1^3 \frac{\mathrm{d}x}{\sqrt{x}(1+x)}$ en effectuant le changement de variable $u = \sqrt{x}$.

   $\mathrm{d}u = \dfrac{1}{2\sqrt{x}}\,\mathrm{d}x = \dfrac{1}{2u}\,\mathrm{d}x \Leftrightarrow 2u\,\mathrm{d}u = \mathrm{d}x$

   Si $x = 1$ alors $u = 1$ et si $x = 3$ alors $u = \sqrt{3}$

   $J = \displaystyle\int_1^{\sqrt{3}} \frac{2u\mathrm{d}u}{u(1+u^2)} = \int_1^{\sqrt{3}} \frac{2\mathrm{d}u}{1+u^2} = 2(\arctan\sqrt{3} - \arctan 1) = 2\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\pi}{6}$

4. Résoudre (E) : $\quad (x^2+1)y' + 2xy = 3x^2 + 1$ sur $I = \mathbb{R}$.

   $(E) \Leftrightarrow y' + \dfrac{2x}{x^2+1}\,y = \dfrac{3x^2+1}{x^2+1}$ car $x^2 + 1 \neq 0$ sur $\mathbb{R}$

   $y_H = C\mathrm{e}^{-\ln(x^2+1)} = \dfrac{C}{x^2+1}$

   MVC : $C' = \dfrac{3x^2+1}{x^2+1}(x^2+1) = 3x^2+1$

   d'où $C = x^3 + x$ et $y_P = \dfrac{x^3+x}{x^2+1}$

   $\boxed{y = \dfrac{x^3+x+C}{x^2+1}, C \in \mathbb{R}}$

Sujet B

1. $(S):\left\{\begin{array}{rcrcrcrl} x & + & 2y & - & z & = & 1 & L_1 \\ -2x & - & 3y & + & 3z & = & -1 & L_2 \\ 3x & - & y & - & z & = & -6 & L_3 \end{array}\right. \Leftrightarrow$

$$\left\{\begin{array}{rcrcrcrl} x & + & 2y & - & z & = & 1 & L_1 \\ & & y & + & z & = & 1 & L_2+2L_1 \\ & - & 7y & + & 2z & = & -9 & L_3-3L_1 \end{array}\right. \Leftrightarrow \left\{\begin{array}{rcrcrcrl} x & + & 2y & - & z & = & 1 & L_1 \\ & & y & + & z & = & 1 & -L_2 \\ & & & & 9z & = & -2 & L_3+7L_2 \end{array}\right.$$

$$\Leftrightarrow \left\{\begin{array}{l} x = -\dfrac{15}{9} = -\dfrac{5}{3} \\ y = \dfrac{11}{9} \\ z = -\dfrac{2}{9} \end{array}\right.$$

2. $f(x) = \dfrac{x^2-2}{x^2} = 1 - \dfrac{2}{x^2}$ sur $I =]0,+\infty[$. $\qquad F(x) = x + \dfrac{2}{x}$

3. Calculer $J = \displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin(x)}{1+\cos^2(x)}\mathrm{d}x$ en effectuant le changement de variable $u = \cos x$.

   $\mathrm{d}u = -\sin x \mathrm{d}x$

   Si $x = 0$ alors $u = 1$ et si $x = \dfrac{\pi}{2}$ alors $u = 0$

   $J = \displaystyle\int_1^0 \frac{-\mathrm{d}u}{1+u^2} = \int_0^1 \frac{\mathrm{d}u}{1+u^2} = \arctan 1 = \frac{\pi}{4}$

4. Résoudre les équations différentielles suivantes :

   $(E): \quad y' \sin x + y \cos x + 1 = 0$ sur $I =]0,\pi[$.

   $(E) \Leftrightarrow y' + \dfrac{\cos x}{\sin x} y = \dfrac{-1}{\sin x}$ car $\sin x \neq 0$ sur $I =]0,\pi[$.

   $y_H = C\mathrm{e}^{-\ln|\sin x|} = \dfrac{C}{\sin x}$ car sur $I =]0,\pi[$, $\sin x > 0$

   MVC $C' = \dfrac{-1}{\sin x} \sin x = -1$ donc $C = -x$ et $y_P = -\dfrac{x}{\sin x}$

   $\boxed{y = \dfrac{C-x}{\sin x}, C \in \mathbb{R}}$