

Correction du Test n° 7

Sujet A

$$1. \sum_{1 \leq i \leq j \leq n} \frac{i}{j} = \sum_{j=1}^n \frac{1}{j} \sum_{i=1}^j i = \sum_{j=1}^n \frac{1}{j} \frac{j(j+1)}{2} = \frac{1}{2} \sum_{j=1}^n (j+1) = \boxed{\frac{n(n+3)}{4}}$$

$$2. \text{ Résoudre le système } (S) : \begin{cases} x - 2y + z = 1 & L_1 \\ -2x + 3y - 3z = -1 & L_2 \\ 3x + y + z = -6 & L_3 \end{cases} \Leftrightarrow$$

$$\begin{cases} x - 2y + z = 1 & L_1 \\ -y - z = 1 & L_2 + 2L_1 \\ 7y - 2z = -9 & L_3 - L_1 \end{cases} \Leftrightarrow \begin{cases} x - 2y + z = 1 & L_1 \\ y + z = -1 & -L_2 \\ -9z = -2 & L_3 + 7L_2 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -\frac{15}{9} = -\frac{5}{3} \\ y = -\frac{11}{9} \\ z = \frac{2}{9} \end{cases}$$

$$3. f(x) = \frac{2+x^2}{x^2} = \frac{2}{x^2} + 1 \text{ sur } I =]0, +\infty[.$$

$$\boxed{F(x) = -\frac{2}{x} + x}$$

Sujet B

$$1. \sum_{1 \leq i \leq j \leq n} \frac{j^2}{i} = \sum_{i=1}^n \frac{1}{i} \sum_{j=1}^i j^2 = \sum_{i=1}^n \frac{1}{i} \frac{i(i+1)(2i+1)}{6} = \frac{1}{6} \sum_{i=1}^n (2i^2 + 3i + 1)$$

$$= \frac{n(n+1)(2n+1)}{18} + \frac{n(n+1)}{4} + \frac{n}{6}$$

$$2. (S) : \begin{cases} x + 2y - z = 1 & L_1 \\ -2x - 3y + 3z = -1 & L_2 \\ 3x - y - z = -6 & L_3 \end{cases} \Leftrightarrow$$

$$\begin{cases} x + 2y - z = 1 & L_1 \\ y + z = 1 & L_2 + 2L_1 \\ -7y + 2z = -9 & L_3 - 3L_1 \end{cases} \Leftrightarrow \begin{cases} x + 2y - z = 1 & L_1 \\ y + z = 1 & -L_2 \\ 9z = -2 & L_3 + 7L_2 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -\frac{15}{9} = -\frac{5}{3} \\ y = \frac{11}{9} \\ z = -\frac{2}{9} \end{cases}$$

$$3. f(x) = \frac{x^2 - 2}{x^2} = 1 - \frac{2}{x^2} \text{ sur } I =]0, +\infty[.$$

$$F(x) = x + \frac{2}{x}$$