Correction du Test n° 8

Sujet A

1. Calculer $J = \displaystyle\int_1^3 \frac{\mathrm{d}x}{\sqrt{x}(1+x)}$ en effectuant le changement de variable $u = \sqrt{x}$.

   $\mathrm{d}u = \dfrac{1}{2\sqrt{x}}\,\mathrm{d}x = \dfrac{1}{2u}\,\mathrm{d}x \Leftrightarrow 2u\,\mathrm{d}u = \mathrm{d}x$

   Si $x = 1$ alors $u = 1$ et si $x = 3$ alors $u = \sqrt{3}$

   $J = \displaystyle\int_1^{\sqrt{3}} \frac{2u\mathrm{d}u}{u(1+u^2)} = \int_1^{\sqrt{3}} \frac{2\mathrm{d}u}{1+u^2} = 2(\arctan\sqrt{3} - \arctan 1) = 2\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\pi}{6}$

2. $\displaystyle\int \mathrm{e}^{(1+2\mathrm{i})x}\mathrm{d}x = \frac{e^x(\cos 2x + \mathrm{i}\sin 2x)(1-2\mathrm{i})}{5} + C.$

3. $\displaystyle\int e^x\cos(2x)\mathrm{d}x = \frac{e^x}{5}(\cos 2x + 2\sin 2x) + C.$

4. $x^2 - 2x - 3 = (x+1)(x-3)$ et $\dfrac{1}{x^2-2x-3} = \dfrac{1}{4}\left(\dfrac{-1}{x+1} + \dfrac{1}{x-3}\right)$

   donc $\displaystyle\int \frac{\mathrm{d}x}{x^2-2x-3} = \frac{1}{4}\left(-\ln|x-1| + \ln|x-3|\right)$

Sujet B

1. Calculer $J = \displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin(x)}{1+\cos^2(x)}\mathrm{d}x$ en effectuant le changement de variable $u = \cos x$.

   $\mathrm{d}u = -\sin x\mathrm{d}x$

   Si $x = 0$ alors $u = 1$ et si $x = \dfrac{\pi}{2}$ alors $u = 0$

   $J = \displaystyle\int_1^0 \frac{-\mathrm{d}u}{1+u^2} = \int_0^1 \frac{\mathrm{d}u}{1+u^2} = \arctan 1 = \frac{\pi}{4}$

2. $\displaystyle\int \mathrm{e}^{(2+\mathrm{i})x}\mathrm{d}x = \frac{e^{2x}(\cos x + \mathrm{i}\sin x)(2-\mathrm{i})}{5} + C.$

3. $\displaystyle\int e^{2x}\cos(x)\mathrm{d}x = \frac{e^{2x}}{5}(2\cos x + \sin x) + C.$

4. $x^2 + 2x - 3 = (x-1)(x+3)$ et $\dfrac{1}{x^2+2x-3} = \dfrac{1}{4}\left(\dfrac{1}{x-1} - \dfrac{1}{x+3}\right)$

   donc $\displaystyle\int \frac{\mathrm{d}x}{x^2+2x-3} = \frac{1}{4}\left(\ln|x-1| - \ln|x+3|\right)$