

Correction du Test n° 7

Sujet A

1.

2. Soit $n \in \mathbb{N}^*$.

$$(a) \quad R_n = \sum_{k=0}^n 4^k 3^{n-k} = 3^n \sum_{k=0}^n \left(\frac{4}{3}\right)^k = 3^n \frac{1 - \left(\frac{4}{3}\right)^{n+1}}{1 - \frac{4}{3}} = 3^n \frac{1 - \left(\frac{4}{3}\right)^{n+1}}{-\frac{1}{3}} =$$

$$-(3^{n+1} - 4^{n+1}) = 4^{n+1} - 3^{n+1}$$

$$(b) \quad P_n = \prod_{k=1}^n \frac{2k}{n+2-k} = 2^n \frac{\prod_{k=1}^n k}{\prod_{j=2}^{n+1} j} = 2^n \frac{1}{n+1} \text{ en posant } j = n+2-k$$

$$3. \quad \prod_{k=1}^n e^{2k} = e^{2 \sum_{k=0}^n k} = e^{n(n+1)}$$

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Sujet B

1.

2. Soit $n \in \mathbb{N}^*$.

$$(a) \quad T_n = \sum_{k=1}^n 2k(k-1) = \sum_{k=1}^n 2k^2 - 2k = 2 \sum_{k=1}^n k^2 - 2 \sum_{k=1}^n k$$

$$T_n = 2 \frac{n(n+1)(2n+1)}{6} - 2 \frac{n(n+1)}{2} = \frac{n(n+1)(2n+1)3n(n+1)}{3} =$$

$$\frac{n(n+1)(2n-2)}{3}$$

$$(b) \quad P_n = \prod_{k=1}^{n-1} \left(-1 + \frac{n}{k}\right) = \prod_{k=1}^{n-1} \frac{n-k}{k} = \frac{\prod_{k=1}^{n-1} n-k}{\prod_{k=1}^{n-1} k} = \frac{\prod_{j=1}^{n-1} j}{\prod_{k=1}^{n-1} k} = 1 \text{ en posant } j = n-k$$

$$3. \quad \sum_{k=0}^n e^{k+1} = e \frac{1 - e^{n+1}}{1 - e}$$