

Correction du devoir maison n° 1

Exercice 1

$$1. \alpha = \frac{115\pi}{6} = \frac{120\pi}{6} - \frac{5\pi}{6} = -\frac{5\pi}{6} [2\pi] \text{ donc } \boxed{\cos(\alpha) = -\frac{\sqrt{3}}{2} \text{ et } \sin(\alpha) = -\frac{1}{2}}$$

$$\beta = -\frac{55\pi}{8} = -\frac{48\pi}{8} - \frac{7\pi}{8} = -\pi + \frac{\pi}{8} [2\pi] \text{ donc } \cos(\beta) = -\cos\left(\frac{\pi}{8}\right) \text{ et } \sin(\beta) = -\sin\left(\frac{\pi}{8}\right)$$

$$\cos^2\left(\frac{\pi}{8}\right) = \frac{1 + \cos\left(\frac{\pi}{4}\right)}{2} = \frac{1 + \frac{\sqrt{2}}{2}}{2} = \frac{2 + \sqrt{2}}{4} \text{ et } \cos\left(\frac{\pi}{8}\right) > 0 \text{ donc}$$

$$\cos\left(\frac{\pi}{8}\right) = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{8}\right) \sin\left(\frac{\pi}{8}\right) \text{ donc } \sin\left(\frac{\pi}{8}\right) = \frac{\sin\left(\frac{\pi}{4}\right)}{2 \cos\left(\frac{\pi}{8}\right)} = \frac{\frac{\sqrt{2}}{2}}{\sqrt{2 + \sqrt{2}}} = \frac{\sqrt{2}}{2\sqrt{2 + \sqrt{2}}}$$

$$\text{Finalement, } \boxed{\cos(\beta) = -\frac{\sqrt{2 + \sqrt{2}}}{2} \text{ et } \sin(\beta) = -\frac{\sqrt{2}}{2\sqrt{2 + \sqrt{2}}}}$$

$$\gamma = \frac{101\pi}{12} = \frac{96\pi}{12} + \frac{5\pi}{12} = \frac{\pi}{2} - \frac{\pi}{12} [2\pi] \text{ donc } \cos(\gamma) = \sin\left(\frac{\pi}{12}\right) \text{ et } \sin(\gamma) = \cos\left(\frac{\pi}{12}\right)$$

Par la même démarche que celle pour trouver le cosinus et le sinus de $\frac{\pi}{8}$, on obtient

$$\boxed{\cos(\gamma) = \frac{1}{2\sqrt{2 + \sqrt{3}}} \text{ et } \sin(\gamma) = \frac{\sqrt{2 + \sqrt{3}}}{2}}$$

$$2. \text{ a) } \cos\left(2x - \frac{\pi}{3}\right) = -\frac{1}{2} = \cos\left(\frac{2\pi}{3}\right) \Leftrightarrow 2x - \frac{\pi}{3} = \frac{2\pi}{3} [2\pi] \text{ ou } 2x - \frac{\pi}{3} = -\frac{2\pi}{3} [2\pi]$$

$$\boxed{\Leftrightarrow x = \frac{\pi}{2} [\pi] \text{ ou } x = -\frac{\pi}{6} [\pi]}$$

$$\text{b) } \cos\left(x + \frac{3\pi}{4}\right) = \sin\left(\frac{x}{4}\right) = \cos\left(\frac{\pi}{2} - \frac{x}{4}\right)$$

$$\Leftrightarrow x + \frac{3\pi}{4} = \frac{\pi}{2} - \frac{x}{4} [2\pi] \text{ ou } x - \frac{3\pi}{4} = -\frac{\pi}{2} + -\frac{x}{4} [2\pi]$$

$$\Leftrightarrow \frac{5}{4}x = -\frac{\pi}{4} [2\pi] \text{ ou } \frac{3}{4}x = -\frac{5\pi}{4} [2\pi] \quad \boxed{\Leftrightarrow x = -\frac{\pi}{5} \left[\frac{8\pi}{5}\right] \text{ ou } x = -\frac{5\pi}{3} \left[\frac{8\pi}{5}\right]}$$

$$3. \text{ a) } \sin\left(\frac{x}{2}\right) > \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Leftrightarrow \frac{x}{2} \in]\frac{\pi}{4} + 2k\pi ; \frac{3\pi}{4} + 2k\pi[, k \in \mathbb{Z}$$

$$\boxed{\Leftrightarrow x \in]\frac{\pi}{2} + 4k\pi ; \frac{3\pi}{2} + 4k\pi[, k \in \mathbb{Z}}$$

$$\text{b) Attention aux valeurs interdites ! Ici } 2x \neq \frac{\pi}{2} + k\pi \Leftrightarrow x \neq \frac{\pi}{4} + \frac{k\pi}{2}, k \in \mathbb{Z}$$

$$\tan(2x) \geq -1 = \tan\left(-\frac{\pi}{4}\right) \Leftrightarrow 2x \in \left[-\frac{\pi}{4} + k\pi ; \frac{\pi}{2} + k\pi\right], k \in \mathbb{Z}$$

$$\Leftrightarrow x \in \left[-\frac{\pi}{8} + \frac{k\pi}{2} ; \frac{\pi}{4} + \frac{k\pi}{2}\right], k \in \mathbb{Z}$$

Exercice 2

$$\begin{aligned} \cos(x) + \sin(2x) = 0 &\Leftrightarrow \cos(x) = -\sin(2x) \\ &\Leftrightarrow \cos(x) = \cos\left(\frac{\pi}{2} + 2x\right) \\ 1. &\Leftrightarrow \exists k \in \mathbb{Z}, x = \frac{\pi}{2} + 2x + 2k\pi \text{ ou } x = -\frac{\pi}{2} - 2x + 2k\pi \\ &\Leftrightarrow \exists k \in \mathbb{Z}, x = -\frac{\pi}{2} + 2k\pi \text{ ou } x = -\frac{\pi}{6} + \frac{2k\pi}{3} \end{aligned}$$

$$\mathcal{S} = \left\{ -\frac{\pi}{2} + 2k\pi, -\frac{\pi}{6} + \frac{2k\pi}{3}, k \in \mathbb{Z} \right\}$$

$$2. \quad 2\cos^2(x) + \cos(x) = 0 \Leftrightarrow \cos(x)(2\cos(x) + 1) = 0$$

Il reste donc à résoudre dans $\mathbb{R}$ $\cos(x) = -\frac{1}{2}$ ou $\cos(x) = 0$,

$$\cos(x) = 0 \Leftrightarrow x = \frac{\pi}{2} + k\pi \quad k \in \mathbb{Z} \text{ et}$$

$$\cos(x) = -\frac{1}{2} \Leftrightarrow x = -\frac{2\pi}{3} + 2k\pi \text{ ou } x = \frac{2\pi}{3} + 2k\pi, k \in \mathbb{Z}.$$

$$\mathcal{S} = \left\{ \frac{\pi}{2} + k\pi, \frac{2\pi}{3} + 2k\pi, -\frac{2\pi}{3} + 2k\pi, k \in \mathbb{Z} \right\}$$