

TDO4

Exo 1 2. $x = \cos\left(\frac{\pi}{5}\right)$ $y = \sin\left(\frac{\pi}{5}\right)$

$$\cos\left(\frac{2\pi}{5}\right) = 2x^2 - 1 \quad \text{et} \quad \sin\left(\frac{2\pi}{5}\right) = 2xy$$

$$\begin{aligned}\sin\left(\frac{3\pi}{5}\right) &= \sin\left(\frac{\pi}{5} + \frac{2\pi}{5}\right) = \sin\left(\frac{\pi}{5}\right)\cos\left(\frac{2\pi}{5}\right) + \sin\left(\frac{2\pi}{5}\right)\cos\left(\frac{\pi}{5}\right) \\ &= y(2x^2 - 1) + 2xy \times x \\ &= 4x^2y - y\end{aligned}$$

$$\frac{3\pi}{5} = \pi - \frac{2\pi}{5} \quad \text{donc} \quad \sin\left(\frac{2\pi}{5}\right) = \sin\left(\frac{3\pi}{5}\right)$$

$$(\Rightarrow) \quad 2xy = 4x^2y - y \quad \text{car } y \neq 0 \quad \left(\sin\left(\frac{\pi}{5}\right) > 0\right)$$

$$(\Rightarrow) \quad 4x^2 - 2x - 1 = 0$$

$$\Delta = 20 \quad x = \frac{2 \pm 2\sqrt{5}}{8} = \frac{1 \pm \sqrt{5}}{4} \quad \text{Or } \cos\left(\frac{\pi}{5}\right) > 0$$

$$\text{donc} \quad \cos\left(\frac{\pi}{5}\right) = \frac{1 + \sqrt{5}}{4}$$

R.q au 1. on a trouvé $\cos\left(\frac{\pi}{5}\right) = \frac{\sqrt{3+\sqrt{5}}}{2\sqrt{2}} = \frac{\sqrt{6+2\sqrt{5}}}{4}$

Or $(1+\sqrt{5})^2 = 6+2\sqrt{5}$ donc on a bien retrouvé le même résultat.

$$\begin{cases} y^2 = 1 - x^2 \\ y > 0 \end{cases} \quad \text{donc} \quad \sin\left(\frac{\pi}{5}\right) = \frac{\sqrt{10-2\sqrt{5}}}{4}$$