Exo 3 $\quad 1 + e^{i\theta} = 2\cos\frac{\theta}{2}\, e^{i\frac{\theta}{2}}$ si $\theta \in [0, \pi]$

donc $\left|2\cos\frac{\theta}{2}\right| = 2\cos\frac{\theta}{2}$

$1 - e^{i\theta} = 2i \sin\frac{\theta}{2}\, e^{i\frac{\theta}{2}}$ idem

$e^{i\theta} + e^{i\theta'} = 2\cos\left(\frac{\theta-\theta'}{2}\right) e^{i\left(\frac{\theta+\theta'}{2}\right)}$

et $\quad 0 \leq \left|\frac{\theta-\theta'}{2}\right| \leq \frac{\pi}{2}$

$z^2 = 1 + e^{i\frac{\pi}{3}} = 2\cos\frac{\pi}{6}\, e^{i\frac{\pi}{6}} = \sqrt{3}\, e^{i\frac{\pi}{6}}$

$\boxed{z = \pm 3^{\frac{1}{4}} e^{i\pi/12}}$

$z^2 + 2e^{i\theta} z - 1 - 2e^{i\theta} = 0$

$\Delta = 4e^{2i\theta} + 4(1 + 2e^{i\theta})$

$= (2e^{i\theta} + 2)^2$

ou $\quad \boxed{\begin{array}{l} z_1 = 1 \quad \text{racine évidente} \\ z_2 = -1 - 2e^{i\theta} \end{array}}$