

T004

Exo4

$$\frac{5\sqrt{2}}{4} + \frac{5i\sqrt{2}}{4} = \frac{5}{2} \left(\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right) = \frac{5}{2} e^{i\frac{\pi}{4}}$$

$$\frac{2}{1-i\sqrt{3}} = \frac{2}{2\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)} = \frac{1}{e^{-i\frac{\pi}{3}}} = e^{i\frac{\pi}{3}}$$

$$1-i = \sqrt{2} e^{-i\frac{\pi}{4}}$$

$$1+i = \sqrt{2} e^{i\frac{\pi}{4}}$$

$$\left(\frac{1-i}{1+i} \right)^7 = \frac{(\sqrt{2} e^{-i\frac{\pi}{4}})^7}{(\sqrt{2} e^{i\frac{\pi}{4}})^7}$$

$$= \frac{\cancel{\sqrt{2}}^7 e^{-7i\frac{\pi}{4}}}{\cancel{\sqrt{2}}^7 e^{7i\frac{\pi}{4}}} = e^{-7i\frac{\pi}{2}} (= i)$$

$$\frac{i(1+i\sqrt{3})^3}{(\sqrt{3}-i)^4} = \frac{e^{i\frac{\pi}{2}} 2^3 \left(e^{i\frac{\pi}{3}}\right)^3}{(2e^{-i\frac{\pi}{6}})^4} = \frac{1}{2} e^{i\left(\frac{\pi}{2} + \pi + \frac{2\pi}{3}\right)} = \frac{1}{2} e^{13i\frac{\pi}{6}} = \frac{1}{2} e^{i\frac{\pi}{6}}$$

$$\text{Exo 5 } (1-i\sqrt{3})^6 e^{3i\frac{\pi}{4}} = (2e^{-i\frac{\pi}{3}})^6 e^{3i\frac{\pi}{4}} = 2^6 \underbrace{e^{-2i\pi}}_1 e^{3i\frac{\pi}{4}} = 2^6 e^{3i\frac{\pi}{4}}$$

$$\left(\frac{1+e^{2i\frac{\pi}{3}}}{\sqrt{6}+i\sqrt{2}} \right)^3 = \frac{\left(2\cos\frac{\pi}{3} e^{i\frac{\pi}{3}} \right)^3}{(2\sqrt{2} e^{i\frac{\pi}{6}})^3} = \frac{2^3 \cos^3\left(\frac{\pi}{3}\right) e^{i\pi}}{2^3 \sqrt{2}^3 e^{i\frac{\pi}{2}}} = \frac{1}{16\sqrt{2}} e^{i\frac{\pi}{2}}$$

$$(1+i)^m = \sqrt{2}^m e^{m i \frac{\pi}{4}} \in i\mathbb{R}^* \Leftrightarrow \frac{m\pi}{4} = \frac{\pi}{2} [\pi] \Leftrightarrow m = 2 + 4k \quad k \in \mathbb{Z}$$