DM 1 - Corrigé.

Exercice 1 : Équations

Pour chaque cas, déterminer la valeur de x :

$$\frac{1}{x} - \frac{1}{6} = \frac{1}{42} \qquad (1)$$

$$\frac{1}{4} - \frac{1}{6} = \frac{1}{x} \qquad (2)$$

$$x^2 - 4x + \mathbf{3} = 0 \qquad (3)$$

$$x^4 - 2x^2 \boldsymbol{-} 6 = 0 \qquad (4)$$

(1) $\frac{1}{x} - \frac{1}{6} = \frac{1}{42}$

$$\Leftrightarrow \frac{1}{x} = \frac{1}{42} + \frac{1}{6} = \frac{6+42}{6 \times 42} = \frac{48}{252} = \frac{24}{126} = \frac{12}{63}$$

$$\Leftrightarrow \frac{1}{x} = \frac{\cancel{3} \times 4}{\cancel{3} \times 21} \quad \text{d'où} \quad \boxed{x = \frac{21}{4}}$$

(2) $\frac{1}{x} = \frac{1}{4} - \frac{1}{6} = \frac{6-4}{6 \times 4} = \frac{2}{24} = \frac{1}{12}$

$$\boxed{x = 12}$$

(3) $x^2 - 4x + 3 = 0 \qquad \Delta = 16 - 12 = 4$

$$x = \frac{4 \pm \sqrt{\Delta}}{2} = \frac{4 \pm 2}{2} = \boxed{3 \text{ ou } 1}$$

(4) posons $X = x^2 \Leftrightarrow x = \pm\sqrt{X}$ si $X > 0$

$$X^2 - 2X + 6 = 0$$

$$\Delta = 4 + 24 = 28$$

$$X = \frac{2 \pm \sqrt{28}}{2} = 1 \pm \sqrt{7}$$

si $X = 1 + \sqrt{7} > 0 \Rightarrow \boxed{x = \pm\sqrt{1+\sqrt{7}}}$

si $X = 1 - \sqrt{7} \Rightarrow \boxed{x = \pm i\sqrt{\sqrt{7} - 1}}$

Exercice 2 : réduction de fraction

Mettre le résultat sous la forme d'une seule fraction :

$$\frac{1}{(n+1)^2} + \frac{1}{n+1} - \frac{1}{n} \qquad (1)$$

$$\frac{a^3 - b^3}{(a-b)^2} - \frac{(a+b)^2}{a-b} \qquad (2)$$

$$\frac{\frac{6(n+1)}{n(n-1)(2n-2)}}{\frac{2n+2}{n^2(n-1)^2}} \qquad (3)$$

(1) $= \frac{n}{(n+1)^2 n} + \frac{(n+1)n}{(n+1)^2 n} - \frac{(n+1)^2}{(n+1)^2 n} = \frac{\cancel{n} + \cancel{n^2} + \cancel{n} - \cancel{n^2} - \cancel{2n} - 1}{(n+1)^2 n}$

$$\boxed{(1) = \frac{-1}{(n+1)^2 n}}$$

(2) $\frac{a^3 - b^3}{(a-b)^2} - \frac{(a+b)^2}{a-b} \times \frac{(a-b)}{(a-b)} = \frac{a^3 - b^3 - (a^2 + b^2 + 2ab)(a-b)}{(a-b)^2}$

$$(2) = \frac{\cancel{a^3} - \cancel{b^3} - \cancel{a^3} - \cancel{b^2 a} - \cancel{2}a^2 b + \cancel{a^2 b} + \cancel{b^3} + \cancel{2}ab^2}{(a-b)^2}$$

$$= \frac{ab^2 - a^2 b}{(a-b)^2} = \frac{ab(\cancel{b-a})}{(a-b)^{\cancel{2}}} = \boxed{\frac{ab}{b-a}}$$

Rq $(a-b)^2 = (b-a)^2$

$$(3)\quad \frac{\dfrac{3\cancel{6}(\cancel{n+1})}{\cancel{n}(\cancel{n+1})(\cancel{2n+1})}\ \cancel{2}(\cancel{n+1})^2}{\dfrac{\cancel{2n+1}\ 2(\cancel{n+1})}{n^{\cancel{2}}(\cancel{n+1})^2}} = \boxed{\frac{3}{2}\,n}$$

Exercice 3 : simplification d'expression

Simplifier au maximum les expressions suivantes :

$$\frac{x}{x-1} - \frac{2}{x+1} - \frac{2}{x^2-1} \quad (1) \qquad\qquad \frac{1}{x} + \frac{x+2}{x^2-4} + \frac{2}{x^2-2x} \quad (2)$$

(1) on Remarque que $(x+1)(x-1) = x^2 - 1$

d'où $\dfrac{x}{x-1} \times \dfrac{x+1}{x+1} - \dfrac{2}{x+1} \times \dfrac{x-1}{(x-1)} - \dfrac{2}{(x+1)(x-1)}$

$$(1) = \frac{x^2 \cancel{+x} - 2x \cancel{+2} \cancel{-2}}{(x+1)(x-1)} = \frac{x^2 - x}{(x+1)(x-1)} = \frac{x(\cancel{x-1})}{(x+1)(\cancel{x-1})}$$

$$\boxed{(1) = \frac{x}{x+1}}$$

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(2) $x^2 - 4 = (x-2)(x+2)$

$$(2) = \frac{1}{x} + \frac{1}{x-2} + \frac{2}{x(x-2)}$$

$$= \frac{x \cancel{-2} + x \cancel{+2}}{x(x-2)} = \frac{2\cancel{x}}{\cancel{x}(x-2)}$$

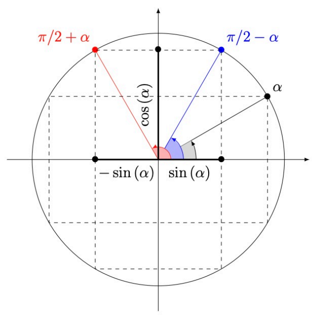
$$\boxed{(2) = \frac{2}{x-2}}$$

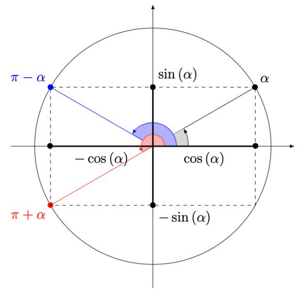
Exercice 4 : trigonométrie

Simplifier les expressions suivantes :

$$\sin(\pi - x) + \cos(\pi/2 + x) \quad (1) \qquad \sin(4x)\cos(5x) - \sin(5x)\cos(4x) \quad (3)$$
$$\sin(\pi/2 - x) + \sin(\pi/2 + x) \quad (2) \qquad \cos(2\omega_1 + \varphi_1)\cos(2\omega_2 + \varphi_2) \quad (4)$$

à développer





(1) $\sin(\pi - x) = \sin(x)$ $\qquad \cos(\pi/2 + x) = -\sin(x)$

$$\boxed{(1) = 0.}$$

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(2) $\sin\left(\frac{\pi}{2}-x\right) = +\cos(x)$

$\sin\left(\frac{\pi}{2}+x\right) = +\cos(x)$

$\Big\}\ (2) = 2\cos(x)$

(3) $\sin(4x)\cos(5x) - \sin(5x)\cos(4x) = \sin(4x-5x)$

$$\boxed{(3) = -\sin(x)}$$

(4) $\cos(2w_1+\varphi_1)\cos(2w_2+\varphi_2)$

$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$

$\Rightarrow 2\cos(a)\cos(b) = \cos(a+b) + \cos(a-b)$

ici $a = 2w_1+\varphi_1$ $\quad b = 2w_2+\varphi_2$

d'où
$$\boxed{(4) = \frac{1}{2}\left[\cos\left\{2(w_1+w_2)+\varphi_1+\varphi_2\right\} + \cos\left\{2(w_1-w_2)+\varphi_1-\varphi_2\right\}\right]}$$