Exercice 1

$\frac{dv}{dt} = a$

j'intègre

$v = at + \cancel{cste}$ → 0 car $v(t=0) = 0$

$v = at$

Soit t_L l'instant où v atteint $v_L = 180$ km.h^{-1}

$t_L = \frac{v_L}{a}$ où $a = 2{,}5$ m.s^{-2}.

j'intègre pour avoir la position.

$x = \frac{1}{2}at^2 + \cancel{cste}$ → 0 car $x(0) = 0$.

$L = \frac{1}{2}at_L^2 = \frac{1}{2}\cancel{a}\frac{v_L^2}{a^{\cancel{2}}} = \frac{1}{2}\frac{v_L^2}{a}$

A.N. $L = \frac{1}{2}\frac{(180/3{,}6)^2}{2{,}5} = 500$ m

Exercice 2

1 $\frac{dv}{dt} = a_0 \Rightarrow v(t) = a_0 t + \cancel{cste}$ → 0 car $v(0) = 0$

j'intègre

à $t = \tau_1$, $\boxed{v_1 = a_0 \tau_1}$

2 $x(t) = \frac{1}{2}a_0 t^2 + \cancel{cste}$ → 0 car $x(0) = 0$

j'intègre

soit d_1 la distance parcourue à τ_1 :

$\boxed{d_1 = x(\tau_1) = \frac{1}{2}a_0\tau_1^2}$

3 pour $t > \tau_1$

$v = v_1 = a_0\tau_1$

d'où $x(t) = v_1 t + cste$

$x(\tau_1) = d_1 = v_1\tau_1 + cste$

$\Leftrightarrow cste = d_1 - v_1\tau_1$

$= \frac{1}{2}a_0\tau_1^2 - a_0\tau_1^2$

$= -\frac{1}{2}a_0\tau_1^2$

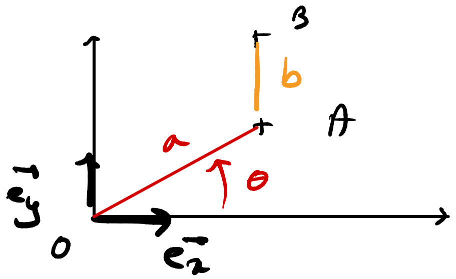
$x(t) = v_1 t - \frac{1}{2}a_0\tau_1^2$

Soit d_2 la distance parcourue au bout de τ_2.

$$d_2 = x(\tau_2) = a_0 \tau_1 \tau_2 - \frac{1}{2} a_0 \tau_1^2$$

$$\boxed{d_2 = a_0 \tau_1 \left(\tau_2 - \frac{\tau_1}{2}\right)}$$

Exercice 3



1) $$\vec{OA} = \left| \begin{array}{l} a\cos(\theta) \\ a\sin(\theta) \end{array} \right.$$

2) $$\vec{OB} = \vec{OA} + \vec{AB}$$

$$= \left| \begin{array}{l} a\cos(\theta) \\ b\sin(\theta) \end{array} \right. + \left| \begin{array}{l} 0 \\ b \end{array} \right. = \left| \begin{array}{l} a\cos(\theta) \\ a\sin(\theta) + b \end{array} \right.$$

3) $$\vec{OA} + \vec{OB} = \left| \begin{array}{l} 2a\cos(\theta) \\ 2a\sin(\theta) + b \end{array} \right.$$

4) $$\vec{OA} - \vec{OB} = -\left(\vec{AO} + \vec{OB}\right) = -\vec{AB} = -b\,\vec{e_y}$$

Exercice 4

1) $\vec{v} = \dfrac{d\,\vec{OM}}{dt} = \left|\begin{array}{l} -a\omega\sin(\omega t) \\ a\omega\cos(\omega t) \\ b \end{array}\right._{cart}$

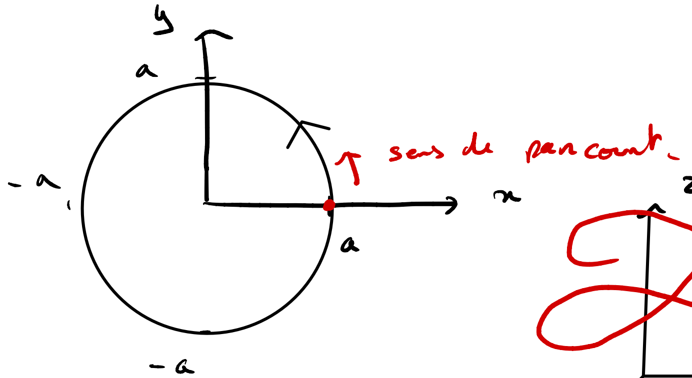
2) $\|\vec{v}\| = \sqrt{a^2\omega^2\sin^2(\omega t) + a^2\omega^2\cos^2(\omega t) + b^2}$

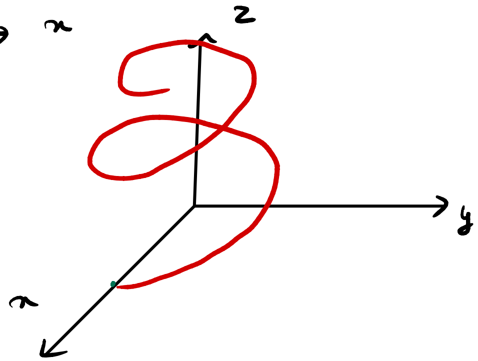
$= \sqrt{a^2\omega^2 + b^2}$

3) $\vec{a} = \dfrac{d\vec{v}}{dt} = \left|\begin{array}{l} -a\omega^2\cos(\omega t) \\ -a\omega^2\sin(\omega t) \\ 0 \end{array}\right._{cart}$

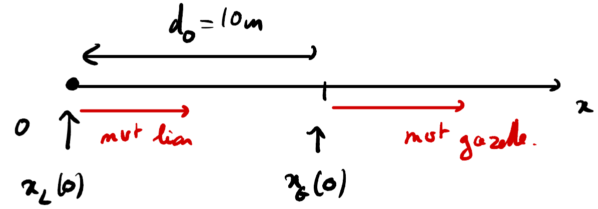
4) $\|\vec{a}\| = \sqrt{a^2\omega^4(\cos^2(\omega t) + \sin^2(\omega t))} = a\omega^2$ ($m.s^{-2}$!)

5)





Exercice 5



$a = \dfrac{dv}{dt} \Rightarrow v(t) = a_0 t + cste$

vitesse initiale du Lion.

$$\begin{cases} v_L(t) = a_L t + v_{L,0} \\ v_G(t) = a_G t + 0 \end{cases}$$

vitesse initiale nulle de la gazelle.

$$\begin{cases} x_L(t) = \frac{1}{2} a_L t^2 + v_{L,0} t + \cancel{cste}\ 0 \\ x_G(t) = \frac{1}{2} a_G t^2 + \cancel{cste}\ d_0 \end{cases}$$

car $x_L(0) = 0$

d_0 car $x_G(0) = d_0$

Le lion attrape la gazelle si $x_L = x_G$, à $t = t_f$.

$\frac{1}{2} a_L t_f^2 + v_{L,0} t_f = \frac{1}{2} a_G t_f^2 + d_0$

$\Leftrightarrow \frac{1}{2}(a_L - a_G)t_f^2 + v_{L,0} t_f - d_0 = 0$ $\qquad \Delta = v_{L,0}^2 + \cancel{\frac{4}{2}}\,2\,(a_L - a_G) d_0$

> 0

d'où $t_{f\_1,2} = \dfrac{-v_{L,0} \pm \sqrt{\Delta}}{(a_L - a_G)}$

La racine négative n'a pas de sens, on ne garde que la racine ≥ 0.

$$\Delta = 5^2 + 2(3-2)\times 10 = 25 + 20 = 45$$

$$t_f = \frac{-5 + \sqrt{45}}{1} \simeq 2\ s \quad (\text{A.N. } 1{,}7\ s).$$

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2 à $t = t_f$

$$x_G(t_f) = \frac{1}{2} a_G t_f^2 + d_0$$

d'où d la distance parcourue est

$$d = x_G(t_f) - \underset{\substack{\uparrow \\ \text{position initiale}}}{d_0} = \frac{1}{2} a_G t_f^2$$

A.N. $\boxed{d = 2{,}9\ m}$

Exercice 6

1 $\frac{dv}{dt} = a_0 \Rightarrow v = a_0 t + c^{ste}$ or $v(t_0) = a_0 t_0 + c^{ste} = 0$

$\Leftrightarrow c^{ste} = -a_0 t_0$

d'où $\boxed{v(t) = a_0 (t - t_0).}$

2 $\frac{dv}{dt} = -kv \Leftrightarrow \frac{dv}{dt} + kv = 0$

sp nulle car second membre nul.

E.D d'ordre 1 : $v(t) = \lambda \exp(-kt) + 0$

C.I à $t = t_0$ $\quad v(t_0) = \lambda \underbrace{\exp(-k t_0)}_{>0} = 0 \Leftrightarrow \lambda = 0$

d'où $\boxed{v(t) = 0}$

3 $\frac{dv}{dt} = -kv + a_0 \Leftrightarrow \frac{dv}{dt} + kv = a_0$

sp pour $v = c^{ste}$ dans

ED d'ordre : $v(t) = \lambda \exp(-kt) + \frac{a_0}{k}$

CI. à $t = t_0$, $v(t_0) = \lambda \exp(-k t_0) + \frac{a_0}{k} = 0$

$\Leftrightarrow \lambda = -\frac{a_0}{k} \exp(k t_0)$

d'où $\boxed{v(t) = +\frac{a_0}{k}\left[1 - \exp - k(t - t_0)\right]}$

4 $\frac{dv}{dt} = -kv^2 \Leftrightarrow \frac{dv}{v^2} = -k\,dt \xrightarrow{\text{j'intègre}} -\frac{1}{v} = -kt + c^{st}$

$$v = \frac{1}{c^{ste} + kt}$$

CI à $t = t_0$, $0 = \dfrac{1}{c^{ste} + k t_0}$ donc $c^{ste} + k t_0 \to +\infty$

$\Leftrightarrow c^{ste} = +\infty$

d'où $\boxed{v(t) = +\infty}$ (pas une sol$^{t°}$ physique).

Exercice 7

$$\vec{a} = \begin{vmatrix} a \cos(\alpha) \\ a \sin(\alpha) \end{vmatrix}$$

$$\vec{b} = \begin{vmatrix} b \sin(\alpha) \\ b \cos(\alpha) \end{vmatrix}$$

$$\vec{c} = \begin{vmatrix} c \cos(\alpha) \\ -c \sin(\alpha) \end{vmatrix}$$

$$\vec{d} = \begin{vmatrix} -d \sin(\alpha) \\ d \cos \alpha \end{vmatrix}$$

Exercice 8

$$\vec{OM} = \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} \frac{1}{2} a_0 t^2 + x_0 \\ - v_0 t \\ z_0 \end{vmatrix}$$

$$\vec{v} = \frac{d\,\vec{OM}}{dt} = \begin{vmatrix} a_0 t \\ - v_0 \\ 0 \end{vmatrix}$$

$$\vec{a} = \frac{d\,\vec{v}}{dt} = \begin{vmatrix} a_0 \\ 0 \\ 0 \end{vmatrix} = a_0 \, \vec{e_x}.$$