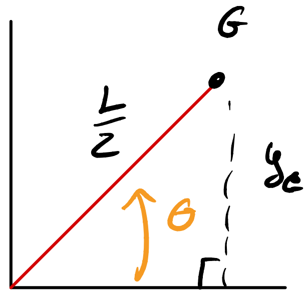
Exercice 1 — Chute d'un arbre

1) $E_c = \frac{1}{2} J \dot\theta^2 = \frac{1}{6} m L^2 \dot\theta^2$

$E_p = + m g y_G = m g \sin(\theta) \frac{L}{2}$

$\uparrow$
y vers le haut



2) $\frac{dE_m}{dt} = P(\cancel{C_{pivot}})$

↳ 0 car la pivot est parfaite

d'où $E_m = c^{ste}$

à l'état initial $\boxed{E_m = 0 + m g \frac{L}{2} \sin(\theta_0)}$

vitesse initiale nulle

3) $\frac{1}{\cancel{6}} \cancel{m} L^{\cancel{2}} \dot\theta^2 + \cancel{m} g \frac{\cancel{L}\,3}{\cancel{2}} \sin(\theta) = \cancel{m} g \frac{\cancel{L}\,3}{\cancel{2}} \sin(\theta_0)$

$\Leftrightarrow \dot\theta^2 = \frac{3g}{L} \left\{ \sin(\theta_0) - \sin(\theta) \right\}$ ← ≥ 0, ok !

d'où $\frac{d\theta}{dt} = \pm \left[\frac{3g}{L} \left\{ \sin(\theta_0) - \sin(\theta) \right\} \right]^{1/2}$

comme $\theta \searrow \Rightarrow \frac{d\theta}{dt} < 0$, on garde la racine négative

d'où $\frac{d\theta}{dt} = - \sqrt{\frac{3g}{L}} \sqrt{\sin(\theta_0) - \sin(\theta)}$

$\boxed{4}$ $$dt = -\sqrt{\frac{L}{3g}} \frac{d\theta}{\sqrt{\sin(\theta_0) - \sin(\theta)}}$$

intègre entre $t = 0$ et $t = T$

⚠ à $t = 0$, $\theta = \theta_0$
à $t = T$, $\theta = 0$

d'où $$T = -\sqrt{\frac{L}{3g}} \int_{\theta_0}^{0} \frac{d\theta}{(\sin\theta_0 - \sin\theta)^{1/2}}$$

$$T = \sqrt{\frac{L}{3g}} \int_{0}^{\theta_0} \frac{d\theta}{(\sin(\theta_0) - \sin\theta)^{1/2}}$$

$$T = 4,315 \times \sqrt{\frac{L}{3g}}$$

A.N $T = 2,5\ s$