Polynômes minimaux

Calculer et factoriser le polynôme minimal de chacune des matrices suivantes (sachant que les racines de ces polynômes sont des entiers relatifs).

$$A_0 = \begin{pmatrix} 2 & 0 & 0 \\ -4 & 10 & -6 \\ -6 & 12 & -7 \end{pmatrix} \quad A_8 = \begin{pmatrix} -6 & -8 & 8 \\ 11 & 15 & -12 \\ 7 & 9 & -6 \end{pmatrix} \quad A_{16} = \begin{pmatrix} 5 & 9 & -8 \\ -1 & -1 & 0 \\ 4 & 8 & -8 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 4 & 4 & -4 \\ -4 & -4 & 4 \\ -2 & -2 & 2 \end{pmatrix} \quad A_9 = \begin{pmatrix} 9 & 10 & -10 \\ -1 & 4 & -2 \\ 4 & 10 & -8 \end{pmatrix} \quad A_{17} = \begin{pmatrix} -4 & 1 & 0 \\ 7 & 6 & -8 \\ 8 & 12 & -13 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 3 & 4 & -4 \\ -4 & -5 & 4 \\ -2 & -2 & 1 \end{pmatrix} \quad A_{10} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad A_{18} = \begin{pmatrix} -7 & -7 & 8 \\ 15 & 19 & -16 \\ 12 & 16 & -12 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} -9 & -12 & 12 \\ 12 & 15 & -12 \\ 6 & 6 & -3 \end{pmatrix} \quad A_{11} = \begin{pmatrix} 7 & 6 & -6 \\ -15 & -20 & 18 \\ -12 & -18 & 16 \end{pmatrix} \quad A_{19} = \begin{pmatrix} 10 & 11 & -10 \\ -3 & 0 & 2 \\ 3 & 7 & -4 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} 1 & 4 & -4 \\ -4 & -7 & 4 \\ -2 & -2 & -1 \end{pmatrix} \quad A_{12} = \begin{pmatrix} 8 & 8 & -8 \\ -2 & 2 & 0 \\ 2 & 6 & -4 \end{pmatrix} \quad A_{20} = \begin{pmatrix} 13 & 15 & -14 \\ -7 & -5 & 6 \\ 1 & 5 & -3 \end{pmatrix}$$

$$A_5 = \begin{pmatrix} 2 & 4 & -4 \\ -4 & -6 & 4 \\ -2 & -2 & 0 \end{pmatrix} \quad A_{13} = \begin{pmatrix} -7 & -6 & 6 \\ 12 & 15 & -14 \\ 9 & 13 & -12 \end{pmatrix} \quad A_{21} = \begin{pmatrix} 9 & 9 & -8 \\ -1 & 3 & 0 \\ 4 & 8 & -4 \end{pmatrix}$$

$$A_6 = \begin{pmatrix} -1 & 0 & 0 \\ -4 & 7 & -6 \\ -6 & 12 & -10 \end{pmatrix} \quad A_{14} = \begin{pmatrix} 0 & 2 & -2 \\ 7 & 11 & -10 \\ 8 & 14 & -13 \end{pmatrix} \quad A_{22} = \begin{pmatrix} -3 & 12 & -8 \\ -7 & 17 & -10 \\ -6 & 12 & -6 \end{pmatrix}$$

$$A_7 = \begin{pmatrix} 2 & 6 & -6 \\ -6 & -10 & 6 \\ -3 & -3 & -1 \end{pmatrix} \quad A_{15} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix} \quad A_{23} = \begin{pmatrix} 12 & 13 & -12 \\ -5 & -2 & 4 \\ 2 & 6 & -3 \end{pmatrix}$$

Solutions :

$\mu_0 = x^2 - 3x + 2 = (x-2)\cdot(x-1)$

$\mu_1 = x^2 - 2x = (x-2)\cdot x$

$\mu_2 = x^2 - 1 = (x-1)\cdot(x+1)$

$\mu_3 = x^2 - 9 = (x-3)\cdot(x+3)$

$\mu_4 = x^2 + 4x + 3 = (x+1)\cdot(x+3)$

$\mu_5 = x^2 + 2x = x\cdot(x+2)$

$\mu_6 = x^2 + 3x + 2 = (x+1)\cdot(x+2)$

$\mu_7 = x^2 + 5x + 4 = (x+1)\cdot(x+4)$

$\mu_8 = x^3 - 3x^2 - 4x + 12 = (x-3)\cdot(x-2)\cdot(x+2)$

$\mu_9 = x^3 - 5x^2 + 2x + 8 = (x-4)\cdot(x-2)\cdot(x+1)$

$\mu_{10} = x^3 - x^2 - 9x + 9 = (x-3)\cdot(x-1)\cdot(x+3)$

$\mu_{11} = x^3 - 3x^2 - 6x + 8 = (x-4)\cdot(x-1)\cdot(x+2)$

$\mu_{12} = x^3 - 6x^2 + 8x = (x-4)\cdot(x-2)\cdot x$

$\mu_{13} = x^3 + 4x^2 - x - 4 = (x-1)\cdot(x+1)\cdot(x+4)$

$\mu_{14} = x^3 + 2x^2 - x - 2 = (x-1)\cdot(x+1)\cdot(x+2)$

$\mu_{15} = x^3 - 3x^2 - 4x + 12 = (x-3)\cdot(x-2)\cdot(x+2)$

$\mu_{16} = x^3 + 4x^2 + 4x = x\cdot(x+2)^2$

$\mu_{17} = x^3 + 11x^2 + 39x + 45 = (x+5)\cdot(x+3)^2$

$\mu_{18} = x^3 - 12x + 16 = (x+4)\cdot(x-2)^2$

$\mu_{19} = x^3 - 6x^2 + 9x - 4 = (x-4)\cdot(x-1)^2$

$\mu_{20} = x^3 - 5x^2 = (x-5)\cdot x^2$

$\mu_{21} = x^3 - 8x^2 + 20x - 16 = (x-4)\cdot(x-2)^2$

$\mu_{22} = x^3 - 8x^2 + 21x - 18 = (x-2)\cdot(x-3)^2$

$\mu_{23} = x^3 - 7x^2 + 11x - 5 = (x-5)\cdot(x-1)^2$