

Corrigé TD développements limités

Exercice 1 (468)

(a)

$$\ln\left(\frac{x^2+1}{x+1}\right) = \ln(1+x^2) - \ln(1+x) = -x + \frac{3}{2}x^2 - \frac{1}{3}x^3 + o(x^3)$$

(b)

$$\ln(1 + \sin x) = x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + o(x^3).$$

(c)

$$\cos(\ln x) = 1 - \frac{1}{2}(x-1)^2 + \frac{1}{2}(x-1)^3 + o((x-1)^3).$$

Exercice 2 (469)

(a)

$$e^{\sqrt{1+x}} = e + \frac{e}{2}x + \frac{e}{48}x^3 + o(x^3).$$

(b)

$$\ln(1 + \sqrt{1+x}) = \ln 2 + \frac{1}{4}x - \frac{3}{32}x^2 + \frac{5}{96}x^3 + o(x^3)$$

(c)

$$\ln(3e^x + e^{-x}) = 2\ln 2 + \frac{1}{2}x + \frac{3}{8}x^2 - \frac{1}{8}x^3 + o(x^3)$$

Exercice 3 (471)

(a)

$$\sin(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{12}\left(x - \frac{\pi}{4}\right)^3 + o\left(\left(x - \frac{\pi}{4}\right)^3\right)$$

(b)

$$\frac{\ln x}{x^2} = (x-1) - \frac{5}{2}(x-1)^2 + \frac{13}{3}(x-1)^3 - \frac{77}{12}(x-1)^4 + o((x-1)^4).$$

(c)

$$\operatorname{sh}x \operatorname{ch}(2x) - \operatorname{ch}x = -1 + x - \frac{1}{2}x^2 + \frac{13}{6}x^3 - \frac{1}{24}x^4 + \frac{121}{120}x^5 + o(x^5).$$

Exercice 4 (472)

(a)

$$(1+x)^{1/x} = e - \frac{e}{2}x + \frac{11e}{24}x^2 + o(x^2)$$

(b)

$$\ln\left(\frac{\sin x}{x}\right) = -\frac{1}{6}x^2 - \frac{1}{180}x^4 + o(x^4)$$

(c)

$$\ln\left(\frac{\operatorname{sh}x}{x}\right) = \frac{1}{6}x^2 - \frac{1}{180}x^4 + o(x^4)$$

Exercice 5

(473)

(a)

$$\ln(1 + e^x) = \ln 2 + \frac{1}{2}x + \frac{1}{8}x^2 + o(x^3)$$

(b)

$$\ln(2 + \sin x) = \ln 2 + \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{24}x^3 + o(x^3)$$

(c)

$$\sqrt{3 + \cos x} = 2 - \frac{1}{8}x^2 + o(x^3)$$

Exercice 6

(474)

(a)

$$\frac{\ln(1+x)}{e^x - 1} = 1 - x + \frac{2}{3}x^2 - \frac{11}{24}x^3 + o(x^3)$$

(b)

$$\frac{\arctan x}{\tan x} = 1 - \frac{2}{3}x^2 + o(x^2)$$

(c)

$$\frac{x-1}{\ln x} = 1 + \frac{1}{2}(x-1) - \frac{1}{12}(x-1)^2 + o((x-1)^2)$$

Exercice 7

(478)

(a)

$$\frac{x - \sin x}{1 - \cos x} = \frac{1}{3}x + \frac{1}{90}x^3 + o(x^3)$$

(b)

$$\frac{\sin x}{\exp(x) - 1} = 1 - \frac{1}{2}x - \frac{1}{12}x^2 + o(x^2)$$

(c)

$$\frac{x\operatorname{ch}x - \operatorname{sh}x}{\operatorname{ch}x - 1} = \frac{2}{3}x + \frac{1}{90}x^3 + o(x^3)$$

Exercice 8

(485)

(a)

$$\ln\left(\frac{x^2+1}{x+1}\right) = -x + \frac{3}{2}x^2 - \frac{1}{3}x^3 + o(x^3)$$

(b)

$$\sqrt{3 + \cos x} = 2 - \frac{1}{8}x^2 + o(x^3)$$

(c)

$$(1+x)^{1/x} = e - \frac{e}{2}x + \frac{11e}{24}x^2 + o(x^2)$$

(d)

$$\frac{\ln(1+x)}{e^x - 1} = 1 - x + \frac{2}{3}x^2 - \frac{11}{24}x^3 + o(x^3)$$